GEOMETRY
MODULE 3 LESSON 3
THE SCALING PRINCIPLE FOR AREA

OPENING EXERCISE
Find the area of shaded region for each figure.

\[(6 \times 19) + \left(\frac{1}{2}\right)(6)(7) = 135\]

\[(8)(9) - [(4 \times 2) + (2 \times 2)] = 60\, cm^2\]

CLASSWORK
Complete the chart in the Exploratory Challenge on page 15 of your workbook. We will look at the first figure together.

- \(\text{Area} = \left(\frac{1}{2}\right)(8)(3) = 12\)
- The dimensions of a similar triangle with a factor of 3 are 24 \(\times\) 9.
- \(\text{Area of the similar triangle} = \left(\frac{1}{2}\right)(24)(9) = 108\)
- Ratio of the Areas is 108: 12 = \(\frac{108}{12} = 9\)

<table>
<thead>
<tr>
<th>(i) Area of Original Figure</th>
<th>Scale Factor</th>
<th>(ii) Dimensions of Similar Figure</th>
<th>(iii) Area of Similar Figure</th>
<th>Ratio of Areas</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 12</td>
<td>3</td>
<td>(24 \times 9)</td>
<td>108</td>
<td>9</td>
</tr>
<tr>
<td>b. 7.5</td>
<td>2</td>
<td>(10 \times 6)</td>
<td>30</td>
<td>4</td>
</tr>
<tr>
<td>c. 20</td>
<td>(\frac{1}{2})</td>
<td>(2.5 \times 2)</td>
<td>5</td>
<td>(\frac{5}{7.5} = \frac{1}{4})</td>
</tr>
<tr>
<td>d. 6</td>
<td>(\frac{3}{2})</td>
<td>(4.5 \times 3)</td>
<td>13.5</td>
<td>(\frac{13.5}{6} = \frac{27}{12} = \frac{9}{4})</td>
</tr>
</tbody>
</table>
After completing the chart, make a conclusion about the relationship between the areas of the original figure and the similar figure with respect to the scale factor between the figures.

The value of the ratio is the square of the scale factor. NOTE: $3^2 = 9; \ 2^2 = 4; \ \left(\frac{1}{2}\right)^2 = \frac{1}{4}; \ \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

DISCUSSION

**The Scaling Principle for Triangles**: If similar triangles $S$ and $T$ are related by a scale factor of $r$, then the respective areas are related by a factor of $r^2$.

Let’s prove the scaling principle for triangles.

- Write two equations: one to represent the area of $S$ and the other to represent the area of $T$.

  $$\text{Area } S = \frac{1}{2}bh \quad \text{Area } T = \frac{1}{2}(rb)(rh)$$

- What does $r$ represent in Triangle $T$?
  
  \[ r \text{ represents the scale factor.} \]

- Create the ratio $\text{Area } T : \text{Area } S$ then simplify.

  \[
  \frac{\text{Area } T}{\text{Area } S} = \frac{\frac{1}{2}(rb)(rh)}{\frac{1}{2}bh} = (r)(r) = r^2
  \]
Can we extend this principle to all polygons? Yes!
Consider the two polygons below. How could we compute the area of each figure?

Each polygon could be broken up into triangles. The sum of areas of those triangles would equal the area for the polygon. We could then compare the ratio of the areas.

The Scaling Principle for Polygons: If similar polygons $P$ and $Q$ are related by a scale factor of $r$, then their respective areas are related by a factor of $r^2$.

PRACTICE
1. Rectangle $A$ and $B$ are similar and are drawn to scale. If the area of rectangle $A$ is 88 mm$^2$, what is the area of rectangle $B$?
   - Find the scale factor. $\frac{30}{16} = 1.875$
   - Apply the Scaling Principle for Polygons.
     $\text{Area scale factor} = (1.875)^2$
     $\text{Area } B = (1.875)^2 \times \text{Area } A$
     $\text{Area } B = (1.875)^2 \times 88 = 309.375 \text{ mm}^2$

2. Complete this exercise in your workbook.
**DISCUSSION**

- Consider a figure that is scaled in only one direction. What can be said about the area of the new figure?

  The area changes by the same factor as the horizontal scale factor, similarly if the figure is scaled only vertically.

Example: Consider a trapezoid with area of 7 cm². If the trapezoid is stretched vertically by a factor of 4, what is the area of the new figure?

\[
\text{Area of new figure} = (\text{original area} \times \text{vertical scale factor}) = 7 \times 4 = 28 \text{ cm}^2
\]

- Consider a figure that is scaled by one factor horizontally and a different factor vertically. What can be said about the area of the new figure?

  The area changes by the same factor as the horizontal and vertical scale factor.

Example: Consider a trapezoid with area of 10 cm². If the trapezoid is stretched vertically by a factor of 6 and compressed by a factor of \(\frac{1}{2}\), what is the area of the new figure?

\[
\text{Area of new figure} = (\text{original area} \times \text{vertical scale factor} \times \text{horizontal scale factor})
\]

\[
\text{Area of new figure} = 10 \times 6 \times \frac{1}{2} = 30 \text{ cm}^2
\]

**ON YOUR OWN**

The small star has an area of 5. The large star is obtained from the small by a factor of 2 in the horizontal direction and a by a factor of 3 in the vertical direction. Find the area of the large star.

\[
\text{Area of Large Star} = 5 \times 2 \times 3 = 30 \text{ sq.units}
\]
SUMMARY

- **The Scaling Principle for Triangles:** If similar triangles $S$ and $T$ are related by a scale factor of $r$, then the respective areas are related by a factor of $r^2$.

- **The Scaling Principle for Polygons:** If similar polygons $P$ and $Q$ are related by a scale factor of $r$, then their respective areas are related by a factor of $r^2$.

- **The Scaling Principle for Area:** If similar figures $A$ and $B$ are related by a scale factor of $r$, then the respective areas are related by a factor of $r^2$.

HOMEWORK

Problem Set Module 3 Lesson 3, page 20

#3, #6, #7, #9, and #10. Show all work in an organized and linear manner.

**DUE: Thursday, February 16, 2017**