**Priority Standard:**

**Strand(s):** RP  
**Topic A:** Proportional Relationships (7.RP.2a August 26 – September 2 (6 days))

<table>
<thead>
<tr>
<th>Previous Grade Standard</th>
<th>Standard(s) for Grade/Course:</th>
<th>Next Grade Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCSS.MATH.CONTENT.6.RP.A.3</td>
<td><strong>CCSS.Math.Content.7.RP.A.2</strong> Recognize and represent proportional relationships between quantities.</td>
<td>CCSS.Math.Content.8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph.</td>
</tr>
<tr>
<td>Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</td>
<td>a) Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.</td>
<td>Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT.6.RP.A.3.A Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</td>
<td></td>
<td>CCSS.Math.Content.8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT.6.RP.A.3.B Solve unit rate problems including those involving unit pricing and constant speed.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT.6.RP.A.3.C - Addressed Later Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In 6th grade the real focus is on equivalent ratios. Students build ratio tables. They begin to analyze the relationships within the table to find other ratios. They learn to find unit rate to compare two sets of data. They also learn to also write them in fractional form. They also learn to plot these points and discover they are a line if they are indeed equivalent and write the equation of the line in terms of $y=kx$ where $k$ is the constant.

In 7th grade instead of talking about unit rate and refer to it as constant of proportionality which could be either $\frac{y}{x}$ or $\frac{x}{y}$. In 8th grade this becomes slope and is strictly defined as vertical change over horizontal change. Students also start learning to write equations with given slopes and points.

**Anchor Problem:**
Mid-Module Assessment #1: Josiah and Tillery have new jobs at YumYum’s Ice Cream Parlor. Josiah is Tillery’s manager. In their first year, Josiah will be paid $14 per hour and Tillery will be paid $7 per hour. They have been told that after every year with the company, they will each be given a raise of $2 per hour. Is the relationship between Josiah’s pay and Tillery’s pay rate proportional? Explain your reasoning using a table.

**Preparing the learner:**
Even though the actual module doesn't begin until September 1, you have 3 built in days to get students up to speed with the 6th grade standards listed above. Pre-assessment is really important here. Students have to know how to use the multiplication chart or a calculator to find equivalent ratios and simplify ratios. They have to be able to find unit rate both as whole and rational values. If students can use a ratio to create a ratio table and plot the points on the coordinate plane in a line they are ready to proceed. Use these three days to shore up the skills and get them ready for topic A in module 1. Almost immediately they change the language and continue on into other forms of proportionality. Make sure students can convert measurement and add and subtract mixed measurements. Also make sure students understand the difference between reductions and enlargements and how to produce them proportionally.

**Big ideas for this topic:**
- Proportionality in tables, graphs, and equations!

**Recall/Skills:**
- I can determine whether values in a ratio table are proportional
- I can determine the unit rate (constant) in a table
- How much fruit should you use with 7 cups of nuts? Is this proportional? Describe how you know.
- Determine the unit rate in the table at right.

<table>
<thead>
<tr>
<th>Serving Size</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cups of Nuts (x)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Cups of Fruit (y)</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>
### Knowledge Packet for Grade 7 Module 1 2016-17: revised 8/10/16

<table>
<thead>
<tr>
<th>I can plot points in the coordinate plane</th>
<th>Plot the data from the chart above to determine whether the function is proportional. How did you make this decision?</th>
</tr>
</thead>
<tbody>
<tr>
<td>I can write an equation using the constant in the form $y = kx$</td>
<td>Write an equation in the form of $y = kx$ from either the ratio table or the graph.</td>
</tr>
</tbody>
</table>

### Making Connections

<table>
<thead>
<tr>
<th>I can solve real life problems involving proportionality</th>
<th>Angel and Jayden were at track practice. The track is 25 kilometers around. Angel ran 1 lap in 2 minutes. Jayden ran 3 laps in 5 minutes.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• How many minutes does it take Angel to run one kilometer? What about Jayden?</td>
</tr>
<tr>
<td></td>
<td>• How far does Angel run in one minute? What about Jayden?</td>
</tr>
<tr>
<td></td>
<td>• Who is running faster? Explain your reasoning.</td>
</tr>
</tbody>
</table>

### Teacher Ideas for Interaction

#### Eureka

In Module 1, students build upon their Grade 6 reasoning about ratios, rates, and unit rates ($6.RP.1$, $6.RP.2$, $6.RP.3$) to formally define proportional relationships and the constant of proportionality ($7.RP.2$). In Topic A, students examine situations carefully to determine if they are describing a proportional relationship. Their analysis is applied to relationships given in tables, graphs, and verbal descriptions ($7.RP.2a$).

*I cannot stress enough how important it is to do some station work using 6th grade module 1 materials. This time will pay off. L1 starts off having students determine the better buy. They did quite a bit of this in the 6th grade and it is a great connection piece to use. Remember the focus is on using double number...*
lines and unit rates.

L2 replaces “constant” with constant of proportionality and the next 3 lessons have students practice completing a ratio table, plotting points on the coordinate plane to discover they form a line and pass through the origin, and write the equation in context over and over. Focus on choosing a few quality problems to dig into deeply. Encourage student to student presentations, writing, and student dialogue around these problems. Use all five days. They will seem like they are the same thing over and over but kids need repetition. After the third day, give an exit ticket and differentiate on the last two days.

**Blended Resources, Personal Learning Resources, Differentiated Learning Resources**

**CCSS Math Resources**

**Common Core stations 7th grade**

**Thinking blocks (excellent resource)**

**Dan Meyer Blog**

**Quia (google quia “proportional relationships) for jeopardy, rags to riches, matching, concentration, or quizzes)**

**Howard County**
- Hair and Nails

**MARS Shell Center**
- Proportion or Not?

**Pre-Assessment Module 1**

**Representations:**

<table>
<thead>
<tr>
<th>Ratio Table</th>
<th>Coordinate Plane</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Equations y=kx</strong></td>
<td><strong>Flour</strong></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

12
9
6
3
0

8
6
4
2
0

12
9
6
3
0

8
6
4
2
0

Flour

Sugar
Vocabulary:

- **Ratio**: A ratio is a statement of how two numbers compare. It is a comparison of the size of one number to the size of another number. All of the lines below are different ways of stating the same ratio. 3:4
- **Rate**: In mathematics, a rate is the ratio between two related quantities. Often it is a rate of change.
- **Unit Rate**: When rates are expressed as a quantity of 1, such as 2 feet per second or 5 miles per hour, they are called unit rates. If you have a multiple-unit rate such as 120 students for every 3 buses, and want to find the single-unit rate, write a ratio equal to the multiple-unit rate with 1 as the second term.
- **Equivalent Ratio**: These ratios are equivalent because they have the same meaning - the amount of water is six times the amount of squash. You can find equivalent ratios by multiplying or dividing both sides by the same number.
- **Proportional To**: (Measures of one type of quantity are proportional to measures of a second type of quantity if there is a number \( k > 0 \) so that for every measure \( x \) of a quantity of the first type the corresponding measure \( y \) of a quantity of the second type is given by \( kx \), i.e., \( y = kx \).)
- **Proportional Relationship**: (A one-to-one matching between two types of quantities such that the measures of quantities of the first type are proportional to the measures of quantities of the second type.)
- **Constant of Proportionality**: (If a proportional relationship is described by the set of ordered pairs that satisfies the equation \( y = kx \), where \( k \) is a positive constant, then \( k \) is called the constant of proportionality; e.g., If the ratio of \( y \) to \( x \) is 2 to 3, then the constant of proportionality is 2/3 and \( y = 2/3 x \).)
- **One-to-One Correspondence**: (Two figures in the plane, \( S \) and \( S' \), are said to be in one-to-one correspondence if there is a pairing between the points in \( S \) and \( S' \), so that, each point \( P \) of \( S \) is paired with one and only one point \( P' \) in \( S' \) and likewise, each point \( Q' \) in \( S' \) is paired with one and only one point \( Q \) in \( S \).)
- **Scale Drawing and Scale Factor**: (For two figures in the plane, \( S \) and \( S' \), \( S' \) is said to be a scale drawing of \( S \) with scale factor \( r \) if there exists a one-to-one correspondence between \( S \) and \( S' \) so that, under the pairing of this one-to-one correspondence, the distance \( |PQ| \) between any two points \( P \) and \( Q \) of \( S \) is related to the distance \( |P'Q'| \) between corresponding points \( P \) and \( Q \) of \( S \) by \( |P'Q'| = r |PQ| \).)

Probing questions:

- What does it mean that the “cost is proportional to weight”?
- How do we know if two quantities are proportional to each other?
- How can we recognize a proportional relationship when looking at a table or a set of ratios?
• Explain how the constant was determined.
• Describe appropriate steps for graphing a set of data to determine proportionality. It may help you to actually make one to think about the steps.
• Describe how to determine proportionality in a table and a graph. Give an example to illustrate your point.
• What is a common mistake a student might make when deciding whether a graph of two quantities shows that they are proportional to each other?
• Compare and contrast the constant of proportionality and the unit rate alike?
• How do we compare ratios when we have varying sizes of quantities?
• How do we know if two quantities are proportional to each other?
• How can we recognize a proportional relationship when looking at tables or a set of ratios?
• Do the x- and y-values need to increase at a constant rate? In other words, when the x- and y-values both go up at a constant rate, does this always indicate that the relationship is proportional?
• When looking at ratios that describe two quantities that are proportional in the same order, do the ratios always have to be equivalent?
• How can you use a table to determine whether the relationship between two quantities is proportional?
• Do the following represent a proportional relationship? Prove it two different ways. Which
**Priority Standard:**

Strand(s): RP

Topic B: Unit Rate and the Constant of Proportionality (7.RP.2b, 7.RP.2c, 7.RP.2d, 7.EE.4a)  September 6-September 9 (4 days)

<table>
<thead>
<tr>
<th>Previous Grade Standard</th>
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</tr>
</thead>
<tbody>
<tr>
<td>CCSS.MATH.CONTENT.6.RP.A.3</td>
<td><strong>This will be the tested priority</strong> Recognize and represent proportional relationships between quantities.</td>
<td>CCSS.Math.Content.8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</td>
</tr>
<tr>
<td>Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</td>
<td>b) Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.</td>
<td>CCSS.Math.Content.8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT.6.RP.A.3.A Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</td>
<td>c) Represent proportional relationships by equations. For example, if total cost ( t ) is proportional to the number ( n ) of items purchased at a constant price ( p ), the relationship between the total cost and the number of items can be expressed as ( t = pn )</td>
<td></td>
</tr>
<tr>
<td>Solve unit rate problems including those involving unit pricing and constant speed.</td>
<td>d) <strong>Explain what a point ((x, y)) on the graph of a proportional relationship means in terms of the situation, with special attention to the points ((0, 0)) and ((1, r)) where ( r ) is the unit rate</strong></td>
<td></td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT.6.RP.A.3.B</td>
<td></td>
<td></td>
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<td>Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.</td>
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<td>CCSS.MATH.CONTENT.6.RP.A.3.C</td>
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<tr>
<td>Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.</td>
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<tr>
<td>CCSS.MATH.CONTENT.6.G.A.1</td>
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<tr>
<td>Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.</td>
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<tr>
<td>CCSS.MATH.CONTENT.6.EE.B.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solve real-world and mathematical problems by writing and solving equations of the form ( x + p = q ) and ( px = q ) for cases in which ( p, q ) and ( x ) are all nonnegative rational numbers.</td>
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</tbody>
</table>
6th graders spend a huge amount of time on determining if things in a ratio are proportional by identifying the constant. This is extended into 7th grade as we begin to see that same constant reappear in both the coordinate plane and in linear equations in the form $y = kx$.

<table>
<thead>
<tr>
<th>Preparing the learner:</th>
<th>Changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make sure to give both the school city pre-assessment AND the formative pre test as well. At this point students have to be able to recognize how to find constant of proportionality in table, graph and equation. Make sure students can divide rationals or have a tool to assist them and that they can plot points on the coordinate plane.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Big ideas for this topic:</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Constant of proportionality</td>
</tr>
<tr>
<td>$y = kx$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Recall/Skills:</th>
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<tbody>
<tr>
<td>- I can identify the constant of proportionality in a table, graph or equation.</td>
</tr>
<tr>
<td>- 1. George is building a fence. He builds his fence at a constant rate of $\frac{1}{3}$ section of fence every $\frac{1}{2}$ hour. At this rate, what fraction represents the section of fence George can build per hour? Express your answer as a fraction.</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>Making Connections</th>
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</thead>
<tbody>
<tr>
<td>- I can identify and interpret in context the point $(1, r)$ on the graph of a proportional relationship where $r$ is the unit rate.</td>
</tr>
<tr>
<td>- Great Rapids White Watering Company rents rafts for $125 per hour. Explain why the point $(0,0)$ and $(1,125)$ are on the graph of the relationship, and what these points mean in the context of the problem.</td>
</tr>
</tbody>
</table>
I can interpret what points on the graph of a proportional relationship mean in terms of the situation or context of the problem, including the point \((0, 0)\).

**Teacher Ideas for Interaction**

**Eureka/EngageNY**

With the concept of ratio equivalence formally defined, students explore collections of equivalent ratios in real world contexts in Topic B. They build ratio tables and study their additive and multiplicative structure (6.RP.3a). They relate ratio tables to equations using the value of a ratio defined in Topic A. In Topic B, students learn that the unit rate of a collection of equivalent ratios is called the *constant of proportionality* and can be used to represent proportional relationships with equations of the form \(y = kx\), where \(k\) is the constant of proportionality (7.RP.2b, 7.RP.2c, 7.EE.4a). Students relate the equation of a proportional relationship to ratio tables and to graphs and interpret the points on the graph within the context of the situation (7.RP.2d). Do both L9-L10, L15 with fidelity. Take your time because this is really, really important that students see these connections. 14 are great applications we don’t have time for. In 3 days, you got make sure they get the connection. Choose one awesome problem to go deeply if you want. Finally in L15, students expand their experience with the coordinate plane (5.G.1, 5.G.2) as they represent collections of equivalent ratios by plotting the pairs of values on the coordinate plane. The Mid-Module Assessment follows Topic B.
### Blended Resources, Personal Learning Resources, Differentiated Learning Resources

**CCSS Math Resources**

**Common Core stations 7th grade**

**Quia** (google “Quia proportional relationships” for jeopardy, rags to riches, matching, concentration, or quizzes)

**Do math together**

### Probing questions:

- How do I find the constant of proportionality?
- What type of relationship can be modeled using an equation in the form \[ y = kx \], and what do you need to know to write an equation in this form?
- Give an example of a real-world relationship that can be modeled using this type of equation and explain why.
- How do you determine which value is \( x \) (independent) and which value is \( y \) (dependent)?
- Which makes more sense: to use a unit rate of “ears of corn per dollar” or of “dollars/cents per ear of corn”? Which one is independent?
- Give an example of a real-world relationship that cannot be modeled using this type of equation and explain why.
- What points are always on the graph of two quantities that are proportional to each other?
- How can you use the unit rate to create a table, equation, or graph of a relationship of two quantities that are proportional to each other?
- How can you identify the unit rate from a table, equation, or graph?
- How do you determine the meaning of any point on a graph that represents two quantities that are proportional to each other?
Focus Standard:

Strand(s): RP
Topic C: Unit Rates (6.RP.2, 6.RP.3b, 6.RP.3d) September 12-September 16 (approximately 5 days)

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<th>Previous Grade Standard</th>
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<tr>
<td>CCSS.MATH.CONTENT.6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</td>
<td><strong>These two are a focus standard, but not priority CCSS.MATH.CONTENT.7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.</strong></td>
<td>CCSS.Math.Content.8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT.6.RP.A.3.B Solve unit rate problems including those involving unit pricing and constant speed.</td>
<td>CCSS.MATH.CONTENT.7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</td>
<td>CCSS.Math.Content.8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</td>
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<td>CCSS.MATH.CONTENT.6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT.6.EE.B.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Changes

6th graders are learning the difference between ratio and fractions. They are learning to find equivalent ratios, determine proportionality, and to use various modeling tools in this process. 7th graders take this to an application model. 6th graders learn to solve one step equations and distribute while 7th graders must become fluent at two step equations and distributing.

8th graders must be able to solve any multistep linear equation. They must be able to determine proportionality in a table, graph, or an equation. They use the proportional relationships to solve real applications. The focus really shifts to understanding rate of change in multiple settings.
Preparing the learner:

Make sure students can convert measurement and add and subtract mixed measurements. This should be on the pretest and probably should include a station prior to the beginning of this unit. Also make sure students understand the difference between reductions and enlargements and how to produce them proportionally.

Big ideas for this topic:

Fractional rates, Constant of proportionality and scale factor, application of proportional relationships

Recall/Skills:

- I can identify proportionality in tables, graphs and equations.
- I can find the constant of proportionality.
- I can find the unit rate when it is a complex fraction.
- I can make a scale drawing with a given scale factor.

- Determine if the quantities of nuts and fruit are proportional for each serving size listed in the table. If the quantities are proportional, what is the constant of proportionality or unit rate that defines the relationship? Explain how you determined the constant of proportionality and how it relates to both the table and graph.

<table>
<thead>
<tr>
<th>Serving Size</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
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<tr>
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<tr>
<td>Cups of Fruit (y)</td>
<td>2</td>
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<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

- If a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.

- Make a scale drawing with a scale factor of \( \frac{1}{10} \) of the rectangle below.
Making Connections

- I can solve real life problems using proportionality
- I understand constant of proportionality and unit rate are the same.

- Angel and Jayden were at track practice. The track is .5 kilometers around.
  - Angel ran 1 lap in 2 minutes.
  - Jayden ran 3 laps in 5 minutes.

  a. How many minutes does it take Angel to run one kilometer? What about Jayden?
  b. How far does Angel run in one minute? What about Jayden?
  c. Who is running faster? Explain your reasoning.

- Compare constant or proportionality and unit rate. Be sure to use an example to illustrate your comparison.

Teacher Ideas for Interaction

Eureka

In Topic C, students extend their reasoning about ratios and proportional relationships to compute unit rates for ratios and rates specified by rational numbers, such as a speed of ½ mile per ¼ hour (7.RP.1). Students apply their experience in the first two topics and their new understanding of unit rates for ratios and rates involving fractions to solve multistep ratio word problems (7.RP.3, 7.EE.4a).

Lesson 11 is really critical. Don’t be so quick to teach the process or answer. In teams, encourage students to commit to who is faster in L11 and prove it. You want them proving it with a variety of models. Make them each present their ideas and encourage respectful argumentation. L12 could be an optional lesson. In this lesson students have to remodel a room (Tile, carpet, baseboard, etc.) If you have the time and your kids need this kind of activity it is very applicable to the real world but will take several days. L13 is almost completely dividing complex fractions so take a look at your kids needs and decide to do a portion of this lesson or skip it. L14 is critical. Heavy application but most importantly make students prove their response with some type of modeling and share out the modeling plans. Don’t be afraid to spend a couple of quality days on these types of problems. And finally, L15 is very important as well as they start plotting fractional points and determining that when points are proportional they do indeed form a line and pass through the origin. L11, 14, and 15 are the focus over the six/seven class days.

Blended Resources, Personal Learning Resources, Differentiated Learning Resources

CCSS Math Resources
- 7.RP.A1
- 7.RP.A3

Common Core stations 7th grade

MARS Shell Center
- A Golden Crown
- Bike Ride
### Probing questions:

- What is a complex fraction?
- How can unit rate be helpful?
- What is a consumer?
- Compare and contrast a commission and a discount markdown.
- What does it mean to be proportional? Explain how you determine proportionality in tables, graphs, and equations.
- What is a scale drawing?
- What are possible uses for enlarged drawings/pictures?
- What are the possible purposes of reduced drawings/pictures?
- Describe what corresponding points or parts means. Be sure to use an example to help support your description.
- How do scale drawings related to rates and ratios?
- Where is the constant of proportionality represented in scale drawings?
- What step(s) are used to calculate scale factors?
- What operation(s) is (are) used to create scale drawings?
Focus

Strand(s): RP  

Topic D: Ratios of Scale Drawings (7.RP.2b, 7.G.1) September 20 – September 28 (7 days)

<table>
<thead>
<tr>
<th>Previous Grade Standard</th>
<th>Standard(s) for Grade/Course:</th>
<th>Next Grade Standard</th>
</tr>
</thead>
</table>
| CCSS.MATH.CONTENT.6.RP.A.3  
Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. | CCSS.MATH.CONTENT.7.G.A.1  
Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. | CCSS.MATH.CONTENT.8.EE.B.5  
Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. |
| CCSS.MATH.CONTENT.6.RP.A.3.B  
Solve unit rate problems including those involving unit pricing and constant speed. |  | CCSS.MATH.CONTENT.8.F.A.1  
Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. |
| CCSS.MATH.CONTENT.6.RP.A.2  
Understand the concept of a unit rate \( \frac{a}{b} \) associated with a ratio \( a:b \) with \( b \neq 0 \), and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is \( \frac{3}{4} \) cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." |  | CCSS.MATH.CONTENT.8.EE.B.6  
Use similar triangles to explain why the slope \( m \) is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation \( y = mx \) for a line through the origin and the equation \( y = mx + b \) for a line intercepting the vertical axis at \( b \). |
| CCSS.MATH.CONTENT.7.RP.A.2  
Recognize and represent proportional relationships between quantities.  
1. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. |  |  |

Changes

- Much time is spent in 6th grade with equivalent ratios and finding unit rate. This has to be fluent. In 7th grade students apply the constant of proportionality to find scale factor in scale drawings.

- The idea of scale drawings moves to similarity in 8th grade. Constant of proportionality moves to slope.
Anchor Problem:

Use your computer to show a map of Island Grove in Greeley, Colorado. The scale of the map tells us that 2 inch on the map is 1 mile. Students find a familiar place using an online map tool (Google earth, mapquest, etc.). *Teachers, try your online tool out before the students to ensure that there is a scale key. Students find the scale that is given to them on the map tool. Discuss what this means in terms of measuring the place that they are observing. What happens when you zoom out, zoom in? Does the scale change?

Create a reduction and enlargement of your original place. What information do you need to have to create your scale drawings? What information do you need to have on your scale drawings?

- So when we were looking at the map of a location of your choice, what happens when you zoom out on the map? What happens when you zoom in?

- Clicking a resizing button on the computer screen will result in an image of Island Grove where the exact same-sized bar now represents 1000 feet.
  a. Do you think the size of Island Grove under the 1000 ft scale will appear smaller or larger than it was under the 1 mile scale?
  b. Draw an accurate picture/map of Island Grove under this new 1000 ft scale.
  c. Was your guess in part a correct? Can you explain why the size of the map changed as it did?

- How would knowing the scale factor for the drawing of the map of your location help in determining if you had a reduction?

- If you had the dimensions of the map for your location how could you find the area? If you knew the actual dimensions for the map area in real life, how could you relate the areas as ratios?

Preparing the learner:
Students have to be able to identify enlargements and reductions and how to create them with scale factor. This should be in your pre-assessment.

Big ideas for this topic:
Recall/Skills:
- I can determine whether a replica is a reduction or an enlargement
- I can identify scale factor in scale drawings

- Is the figure at right a reduction or enlargement? Explain how you know.

Look at the triangles in the diagram below:

Some of the shapes are enlargements of triangle A.

For each of the other shapes, decide if it is an enlargement of A, and find the scale factor if it is.
- Shape B: Enlarge...
- Shape C: Enlarge...
- Shape D: Enlarge...
- Shape E: Enlarge...
- Shape F: Enlarge...

Work out where the final point in the enlarged shape will go, and the scale factor of the enlargement.
Knowledge Packet for Grade 7 Module 1 2016-17: revised 8/10/16

- I can make a scale drawing given a scale factor
- I can find the area of an scale drawing given the scale factor

- A 1-inch length in the scale drawing corresponds to actual length of 12 feet in the room.
  
  - Describe how the scale factor can be used to determine the actual area of the dining room
  - Find the actual area of the dining room.
  - Can a rectangular table that is 7 feet long and feet wide fit into the narrow section of the dining room. You must justify your answer with math.

Making Connections
- I understand that constant of proportionality in a scale drawing is the same as scale factor
- I understand that enlargements are created with the product of a length and value greater than one and reductions are created with the product of a length and a value between 0 and 1.
- I can create my own scale drawing

- Compare proportionality in a scale drawing and scale factor. Use an example to support your comparison.
- If two figures have a scale factor of 1:3 is it an enlargement or reduction? Explain your response.
- Variable possibilities

Teacher Ideas for Interaction

Eureka

In the final topic of this module, students bring the sum of their experience with proportional relationships to the context of scale drawings (7.RP.2b, 7.G.1). Given a scale drawing, students rely on their background in working with side lengths and areas of polygons (6.G.1, 6.G.3) as they identify the scale factor as the constant of
Knowledge Packet for Grade 7 Module 1 2016-17: revised 8/10/16

proportionality, calculate the actual lengths and areas of objects in the drawing, and create their own scale drawings of a two-dimensional view of a room or building. The topic culminates with a two-day experience of students creating a new scale drawing by changing the scale of an existing drawing. Later in the year, in Module 4, students will extend the concepts of this module to percent problems.

L16 could totally be turned into a station in a station rotation at the beginning of the module. This is where enlargement and reductions have to become fluent. Students must know an enlargement results in a product of a number and a scale factor greater than one while a reduction results in a product of a number and scale factor between 0 and 1. Negative numbers do not affect scale factor. Remind students these reflect the figures. L17 solidifies that the constant of proportionality and the scale factor are one in the same. I think there might be better resources out there but the objective is critical.

L18 is where I would begin teaching once you KNOW from a formative assessment that students understand how to reduce and enlarge. We are back to scale drawings. Lots of practice here and worthwhile time. L19 is optional as it asks students do the scale drawing and then compare the areas. This is an exposure topic in 7th grade. L20-21 are project oriented. You could differentiate the various lessons for students with the minimum being “I can make a scale drawing given a scale” to scaling projects.

Blended Resources, Personal Learning Resources, Differentiated Learning Resources

CCSS Math Resources

Common Core stations 7th grade

Quia (google Quia scale drawings for jeopardy, rags to riches, matching, concentration, or quizzes)

Post Assessment Module 1

Common Assessment Module 1

Probing questions:
- What is the difference between an original picture and a scale drawing?
- How do you know if a scale drawing is a reduction or an enlargement of the original image?
- How can we prove that a reduction has a scale factor less than 1 and an enlargement has a scale factor greater than 1?
- How does the size of the area you are looking at affect your scale?
- How can we relate area to ratios?
- What materials and information do you need to create a scale drawing?
- Why would you need to produce a scale drawing of a different scale?
- How does your scale drawing change when a new scale factor is presented?
- If you made a scale drawing of your map location to include in a party invite why would you need to change to a different scale?
**Priority Standard:**

Strand(s): NS

**Topic A: Addition and Subtraction of Integers and Rational Numbers (7.NS.A.1)**

<table>
<thead>
<tr>
<th>Previous Grade Standard</th>
<th>Standard(s) for Grade/Course:</th>
<th>Next Grade Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCSS.MATH.CONTENT.6.NS.C.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values</td>
<td><strong>CCSS.MATH.CONTENT.7.NS.A.1</strong> Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.</td>
<td></td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT 6.NS.6a Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line</td>
<td>a) Describe situations in which opposite quantities combine to make 0.</td>
<td></td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT 6.NS.6c Find and position integers and other rational numbers on a horizontal or vertical number line.</td>
<td>b) Understand p + q as the number located a distance</td>
<td>q</td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT 6.NS.7a Interpret statements of inequality as statements about relative position of two numbers on a number line.</td>
<td>c) Understand subtraction of rational numbers as adding the additive inverse, p - q = p + (-q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.</td>
<td></td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT 6.NS.7b Write, interpret, and explain statements of order for rational numbers in real-world contexts.</td>
<td>d) Apply properties of operations as strategies to add and subtract rational numbers.</td>
<td></td>
</tr>
</tbody>
</table>

**Changes**

From understanding positive and negative numbers in real-world contexts to adding and subtracting rational numbers on a number line.

New: rational numbers, absolute value, additive inverse, properties of operations

**Changes**

From an understanding of rational numbers to irrational numbers. From using a horizontal or vertical number line diagram to comparing the size of irrational numbers on a number line diagram.

New: irrational numbers, decimal expansion, estimating the value of expressions
**Anchor Problem**

Why do kids need to know how to add and subtract integers/rational numbers? Find a connection that makes them see the need. Perhaps it is talking to their parent about how they have to add and subtract positive and negative numbers. Perhaps it is a really cool video from “When am I ever going to use this.” Or you can have students think about and discuss what purchases and transactions they have made the past month. Where are they spending a lot of their money, where are they getting their money from? Have students create a personal transaction table and log for the remainder of the module.

Create a classroom economy: Pay students for consistent attendance, positive behavior, and excellent achievement. Students then pay for "extras" in the classroom: bathroom break, writing utensils, paper, technology, seating preference, drinking fountain...

Get creative and make sure to make time to make the connection. Use the summary time each day to refer back to the anchor problem to keep the connection alive throughout. Be creative!!

- How can you end up zero school bucks? How does earning and spending school bucks relate to positive and negative numbers?
- Which number line would be the most appropriate type of number line to help in figuring out how many school bucks you have? Why or Why not?
- If your School buck account is negative will it always stay negative? How can it change to positive? What keeps it negative?
- How can distance on a number line be used to understand what is going on with your school bucks account?

**Preparing the learner:**

Make sure to give a pretest and use the days provided prior to the unit to hone skills needed. For sure students need to have a solid understanding of the number line. Make sure there is one in the room, on the floor, or available individually for students. Make sure you include something on the pretest to make sure students can model all operations of positive integers. You could also include operations of fractions. These would make nice differentiated stations prior to starting the module.

**Big Ideas for the topic:**

**Adding and subtracting integers**

<table>
<thead>
<tr>
<th>Recall/Skills: Level 1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I can describe a situation in which opposites quantities combine to make zero.</td>
<td>Credit/debit The bank credited me with $12 interest but then I used my debit card and spent $12 so my balance is 0. Other possible pairs: gain/loss, deposit/withdrawal, grow/decay, warmer/colder, above sea level/below sea level, rise/fall</td>
</tr>
</tbody>
</table>
• I can model addition and subtraction of integers

For each expression in the table, select which number line model, if any, can be used to represent the expression.
Select all appropriate cells in the table.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Number Line 1</th>
<th>Number Line 2</th>
<th>Number Line 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2 + 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>−2 − 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>−2 − (−4)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>−4 + 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>−4 − (−2)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Neither number line model can be used to represent the expression.

1. Which expression is equivalent to 2.2 − 2.5?
   - A. 2.5 − 2.2
   - B. 2.2 + 2.5
   - C. 2.2 + (−2.5)
   - D. 2.2 − (−2.5)
Knowledge Packet for Grade 7 Module 2

- I can add and subtract integers

- I can add and subtract rational numbers

- $-16 + 22 =$

- $-8 + 1\frac{2}{3} =$

Making Connections-Level II (Interacting with the math)

- I can describe a situation in which opposites quantities combine to make zero.

- I can interpret sums of rational numbers by describing real-world contexts.

Formative Assessment Options

Which situations would result in a temperature change of 0°? Select two that are correct.

- A. The temperature rose 9° from 9:00 a.m. to 10:00 a.m. The temperature then increased 9° from 10:00 a.m. to 11:00 a.m.
- B. The temperature increased 10° from 5:00 p.m. to 6:00 p.m. The temperature then decreased 10° from 6:00 p.m. to 7:00 p.m.
- C. The temperature dropped 6° from 7:00 p.m. to 8:00 p.m. The temperature then fell 8° from 8:00 p.m. to 9:00 p.m.
- D. Over a 3 hour period of time the temperature dropped 3° each hour.
- E. The temperature decreased 4° from 1:00 p.m. to 2:00 p.m. The temperature then rose 4° from 2:00 p.m. to 3:00 p.m.

Which of these story problems describes the sum $19 + (-12)$? Check all that apply. Show your work to justify your answer.

- X Jared’s dad paid him $19 for raking the leaves from the yard on Wednesday. Jared spent $12 at the movie theater on Friday. How much money does Jared have left?
- X Jared owed his brother $19 for raking the leaves while Jared was sick. Jared’s dad gave him $12 for doing his chores for the week. How much money does Jared have now?
- X Jared’s grandmother gave him $19 for his birthday. He bought $8 worth of candy and spent another $4 on a new comic book. How much money does Jared have left over?
I can identify the solution of an addition or subtraction problem as positive or negative without actually computing it.

I can find the distance between two points by finding the difference of the absolute value.

Circle the integer with the greater absolute value. Decide whether the sum will be positive or negative without actually calculating the sum.

a. \(-1 + 2\) positive

b. \(5 + (-9)\) negative

c. \(-6 + 3\) negative

d. \(-11 + 1\) negative

Two 7th grade students, Monique and Matt, both solved the following math problem:

If the temperature drops from 7°F to -17°F, by how much did the temperature decrease?

The students came up with different answers. Monique said the answer is 24°F, and Matt said the answer is 10°F. Who is correct? Explain, and support your written response with the use of a formula and a vertical number line diagram.

Teacher Ideas for Interaction

Eureka

In Topic A, students return to the number line to model the addition and subtraction of integers (7.NS.A.1). They use the number line and the Integer Game to demonstrate that an integer added to its opposite equals zero, representing the additive inverse (7.NS.A.1a, 7.NS.A.1b). Their findings are formalized as students develop rules for adding and subtracting integers, and they recognize that subtracting a number is the same as adding its opposite (7.NS.A.1c). Real-life situations are represented by the sums and differences of signed numbers. Students extend integer rules to include the rational numbers and use properties of operations to perform rational number calculations without the use of a calculator (7.NS.A.1d).
Lessons 1-7 all have a heavy modeling aspect: Primarily there are two types of modeling used to teach sums and differences of integers in this topic. The first is via the number line where students associate negative with movement to the left on the number line and positive is associated with movement to the right on the number line (see representations above). Some frontloading may be required if their understanding of the number line is not strong. Use the pre-test to ascertain this. Make sure students are seeing both horizontal and vertical number lines as this sets up the coordinate plane as two number lines, one vertical and one horizontal. The second use of modeling is through the use of a standard deck of cards where red cards represent negative values as they do on a spreadsheet, and black cards represent positive values. The cards are used both in a game fashion that is explained in L1 and then expands as you move through the first 6 lessons and separately to make specific connections.

Think about how to create stations around the probing questions and critical connections. Visualize how you can take part of a lesson and make it a station focus with its own exit ticket. How can you use the modeling to push them toward discovering the algorithms? Think about which key problems in the lessons contribute to leading students to a generalization or move them toward efficient computation and create a station around them. And finally, think about the contextual piece. Where are students going to struggle and set up your stations around these misconceptions or areas of struggle?

Critical connections:
A critical connection happens in L4 where students spend time identifying a solution to a sum as either positive or negative without actually computing. This is a critical piece in helping students discover the rules for addition as they recognize models help but are inefficient. Help lead them towards the concept of what happens when signs are the same and what happens when signs are different and keep asking, how can we do this more efficiently? Don’t give them the rules. Allow them to figure it out even if the language is not correct and then solidify the rules in a summary piece. All of the work in lessons 1-4 is leading to these generalizations:

RULE: Add integers with the same sign by adding the absolute values and using the common sign.

RULE: Add integers with opposite signs by subtracting the absolute values and using the sign of the integer with the greater absolute value.

Another critical connection happens in L5 when they discover that subtraction is really changing directions, or adding the opposite. Allow the discovery and push students to generalize while you are fixing misconceptions.

The Rule of Subtraction: Subtracting a number is the same as adding its additive inverse (or opposite).

L8 “The opposite of a sum is the sum of its opposites.” This is another important connection piece for helping students understand –(a+b) and distributing by a negative.

L9 can be omitted if time is a factor.

Blended Resources, Personal Learning Resources, Differentiated Learning Resources

CCSS Math Resources

Common Core stations 7th grade
**Howard County**
- Adding and Subtracting Rationals
- Time Zone Challenge

**Quia (google quia adding and subtracting integers for jeopardy, rags to riches, matching, concentration, or quizzes)**

**Pre-Assessment Module 2**

**Representations:**
- Equations
- Expressions
- Integer Game (see L1 in module 2 of EngageNY for instructions - uses playing cards to represent positive and negative integers)
- Number Line
- Tape Diagram
- Counters

**Vocabulary:**
- **Additive Identity** (The additive identity is 0.)
- **Additive Inverse** (The additive inverse of a real number is the opposite of that number on the real number line. For example, the opposite of $-3$ is 3. A number and its additive inverse have a sum of 0.)
- **Break-Even Point** (The break-even point is the point at which there is neither a profit nor loss.)
- **Distance Formula** (If $p$ and $q$ are rational numbers on a number line, then the distance between $p$ and $q$ is $|p - q|$.)
- **Loss** (A decrease in amount, as when the money earned is less than the money spent.)
- **Multiplicative Identity** (The multiplicative identity is 1.)
- **Profit** (A gain, as in the positive amount represented by the difference between the money earned and spent)
- **Repeating Decimal** (The decimal form of a rational number, for example, $\frac{1}{3} = 0.\overline{3}$)
- **Terminating Decimal** (A decimal is called terminating if its repeating digit is 0.)
- **Absolute Value** (the positive distance between two points)
- **Associative Property** (of Multiplication and Addition) $a + (b + c) = (a + b) + c$
- **Commutative Property** (of Multiplication and Addition) $a + b = b + a$
- **Credit** (add an amount of money to an account)
- **Debit** (the removal of money from an account)
- **Deposit** (the addition of money to an account)
Knowledge Packet for Grade 7 Module 2

- Distributive Property (of Multiplication Over Addition) \(2(x + 3) = 2x + 6\)
- Expression (2 or more numbers or variables including 1 or more operators)
- Equation (2 expressions with an equal sign)
- Integer (All whole numbers and their opposites)
- Inverse (Relating to a mathematical operation whose nature or effect is the opposite of another operation.)
- Multiplicative Inverse (reciprocal ie: \(\frac{2}{3}, \frac{3}{2}\))
- Opposites (a positive and negative number with the same absolute value)
- Overdraft (a fee charged for over spending an account)
- Positives (All values to the right of 0 (greater than 0) on the number line)
- Negatives (All values to the left of 0 (less than 0) on the number line)
- Rational Numbers (any number which can be written as a fraction as long as the denominator is not zero)
- Withdraw

Probing Questions:
- How do you create a "zero pair"? (Both on a number line and with cards)
- What are similarities and differences positive and negative numbers? (L1 Venn Diagram)
- How is a number line useful in adding integers? (Students should talk about direction, ability to count, and ease of change direction)
- When would you choose to use a vertical number line instead of a horizontal? Horizontal over vertical? (Temperature, sea level work best vertically whereas credit and debit might work better horizontally. Make sure students practice both because it sets up the coordinate plane as just a horizontal and vertical number line)
- Specifically around "the integer game"
- How did selecting positive value cards change the value of your hand?
- How did selecting negative value cards change the value of your hand?
- How did discarding positive value cards change the value of your hand?
- How did discarding negative value cards change the value of your hand?
- What operation reflects selecting a card?
- What operation reflects discarding or removing a card?
- Based on the game, can you make a prediction about what happens to the result when:
  - Subtracting a positive integer.
  - Subtracting a negative integer.
- Will the sum always be a negative if you add a positive and a negative integer?
- If you add two negative numbers, does the sum get higher or lower?
- Can you develop a set of rules for adding integers?
- Can you develop a set of rules for subtracting integers?
- Is the distance between -5 and 4 the same as the distance between -4 and 5?
<table>
<thead>
<tr>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can distance be negative?</td>
</tr>
<tr>
<td>How do you represent a fraction on a number line?</td>
</tr>
<tr>
<td>How can the number line help with sums and differences of rational numbers</td>
</tr>
</tbody>
</table>
### Focus Standard:

**Previous Grade Standard**

| CCSS.MATH.CONTENT.6.NS.A.1 | Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi? |

| Focus Standard |

**Standard(s) for Grade/Course:**

- **CCSS.MATH.CONTENT.7.NS.A.2**
  - **Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.**
  
  a) Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
  
  b) Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then -(p/q) = (-p)/q = p/(-q). Interpret quotients of rational numbers by describing real-world contexts.
  
  c) Apply properties of operations as strategies to multiply and divide rational numbers.
  
  d) Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

| Next Grade Standard |

| CCSS.MATH.CONTENT.8.NS.A.1 | Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. |

| CCSS.MATH.CONTENT.8.NS.A.2 | Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π2). |
### Anchor Problem:

**Preparing the learner:** Make sure students have multiplication tables available at all times as well as number lines. We can’t give them a calculator just yet. Not until they have mastered the rules. The pretest should contain something where students are asked to model all operations with positive numbers.

### Big ideas for this topic:

#### Recall/Skills:
- To multiply and divide rational numbers
- If $p$ and $q$ are integers, then $-(p/q) = (-p)/q = p/(-q)$.
- To write fractions as decimals

#### Making Connections
- Multiplication is repeated addition; Division is repeated subtraction.
- Students will use modeling to discover the rules for multiplication and division (products and quotients of same sign (or even terms) is positive; products and quotients unlike sign (or odd negative terms) are negative)
- Students will solve problems in context involving multiplication and division of rational numbers

#### Teacher Ideas for Interaction
- Represent $(3)(-3)$ with counters and find the product
- Give the sign of the following problems without calculating the answer:
  - $(-2)(-2)(-2)(-234)(-56)$
  - $(-1)(5)(-5)(8)$
  - $(-8)(-4)$
- The water level in Ricky Lake changes at an average of $-\frac{7}{16}$ inch every 3 years.
Students develop the rules for multiplying and dividing signed numbers in Topic B. They use the properties of operations and their previous understanding of multiplication as repeated addition to represent the multiplication of a negative number as repeated subtraction (7.NS.A.2a). Students make analogies to the Integer Game to understand that the product of two negative numbers is a positive number. From earlier grades, they recognize division as the inverse process of multiplication. Thus, signed number rules for division are consistent with those for multiplication, provided a divisor is not zero (7.NS.A.2b). Students represent the division of two integers as a fraction, extending product and quotient rules to all rational numbers. They realize that any rational number in fractional form can be represented as a decimal that either terminates in 0s or repeats (7.NS.A.2d). Students recognize that the context of a situation often determines the most appropriate form of a rational number, and they use long division, place value, and equivalent fractions to fluently convert between these fractions and decimal forms. Topic B concludes with students multiplying and dividing rational numbers using the properties of operations (7.NS.A.2c).

The integer game is particularly useful in helping students identify the rules for addition and subtraction of integers (2 sided color counters do the same). We don’t feel it is the best modeling tool when it comes to multiplication and division. Number lines work best for these rules. Lessons 10-12 help develop the rules for multiplication and division of integers; lesson 15-16 extends this to all rational numbers. Definitely do the coordinate plane activity in L11! Heavy focus on modeling with counters to discover the rules and then lots of computational practice. Lessons 13-14 concentrate on helping students write rational numbers as decimals (preparing them for 8th grade irrational numbers). Teachers may use partial quotients, long division, or equivalent ratios to accomplish this.

**Blended Resources, Personal Learning Resources, Differentiated Learning Resources**

**CCSS Math Resources**

**Common Core stations 7th grade**

Quia (google quia all operations integers for jeopardy, rags to riches, matching, concentration, or quizzes)
Probing questions:

- 3×5 can be represented as 5+5+5, which has a value of 15. If we use the same definition for multiplication, what should the value of 3×(-5) be?
- If the distributive property works for both positive and negative numbers, what expression would be equivalent to 3× (5+(-5))?
- If we use the fact that 5+ (-5) =0 and 3×5=15, what should the value of 3× (-5) be?
- Use what you know from parts (3) and (4). If we can multiply signed numbers in any order, what should the value of (-5)×3 be?
- If the distributive property works for both positive and negative numbers, what expression would be equivalent to (-5)×(3+(-3))?
- Use what you know from parts (3), (4), and (5). What should the value of (-5)×(-3) be?
- How is division of integers regrouping numbers?
- How is this connected to integers?
- How can we write a rule based on our observations from repeated addition or regrouping to divide?
- How can we explain why a negative integer times a negative integer equals a positive value?
- Is multiplying negative integers similar or different than adding negative integers?
- What different ways can we write -3/4? Why?
- Can all fractions be made into decimals?
- Can all decimals be made into fractions?
• What is the difference between a terminating and non-terminating decimal
• How does order of operations apply to integer rules?
• Why can it be helpful to decompose numbers when multiplying or dividing?
• If given the option, would we rather multiply 4/5 by 10 or 12?
• How is multiplying 2 x (3 1/2) like 2 times 3 and 2 times 1/2
### Focus/Priority Standard:

#### Strand(s): NS, EE

**Topic C:** Applying Operations to Expressions and Equations (7.NS.A.3, 7.EE.B.4)  October 25 – November 4 (9 days)

<table>
<thead>
<tr>
<th>Previous Grade Standard</th>
<th>Standard(s) for Grade/Course:</th>
<th>Next Grade Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCSS.MATH.CONTENT.6.NS.B.3</td>
<td><strong>CCSS.MATH.CONTENT.7.NS.A.3</strong> Solve real-world and mathematical problems involving the four operations with rational numbers.</td>
<td>CCSS.MATH.CONTENT.8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.</td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT.6.EE.B.6</td>
<td><strong>CCSS.MATH.CONTENT.7.EE.B.4</strong> Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.</td>
<td>CCSS.MATH.CONTENT.8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.</td>
</tr>
</tbody>
</table>

#### Changes

6th graders learn the inverse process of solving one step equations. The inverse process is based on “undoing” the order of operation. Ie: \[ 2x = 6 \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>inverse</th>
<th>[ x = 3 ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Times 2</td>
<td>Divide by 2</td>
<td>[ 6 \div 2 = 3 ]</td>
</tr>
<tr>
<td>Equals 6</td>
<td>Start with 6</td>
<td></td>
</tr>
</tbody>
</table>

This notion is extended into 2 step equations and must become fluent at this level.

**Anchor Problem:**

---

Students move from solving 2 step equations to solving multi-step equations that requires simplifying before solving.
Preparing the learner:

Make sure to give the pretest early enough to use the three days prior to the topic beginning to hone needed skills. Students must be proficient in the order of operation so spend the time making sure they are ready. They must also have to be able to identify inverse operations.

Start with a scenario like: I started my day giving my boyfriend $10 for lunch. Then I stopped by 7-11 and spent $2.13 on coffee and donuts and then loaned a friend $2. I still have $7.67 in my pocket. How much did I start the day with? Students will intuitively start solving backwards.

Big ideas for this topic:

Solving 2 step equations – most people tested over module 2 and moved this topic to module 3. Your choice.

Recall/Skills:

- I can solve two step equations.
- Solve: \( \frac{5}{4}n + 5 = 20 \)

Making Connections:

- I can write an equation to solve a real life problem using algebra.
- Amie had $26 dollars to spend on school supplies. After buying 10 pens, she had $14.30 left. How much did each pen cost?

Teacher Ideas for Interaction

Eureka

L17 uses tape diagrams to solve equations. Use this! L18 focuses on evaluating. It is in good context there are other resources listed below. The most important focus is that student know how to plug their answer back in and check to see if they did it correctly. We recommend the use of calculators to do this. In other words, “how do you know if a value is a solution to the equation?”. L19 focuses specifically on the distributive property and equivalent expressions. There is an activity in the lesson for this.

We recommend skipping lessons 20-23 and using the inverse method.

Solve \( 2x + 3 = 7 \)

How was this equation constructed?

1. (variable to solve) \( x \)
2. (order of operation says multiply before adding) times 2
3. add 3
4. equals 7

Now identify all of the inverse operations and solve it backwards

Begin with 7 and subtract 3 since the inverse of +3 is -3. 7-3=4
Now take 4 divided by 2 since the inverse of multiplication is division: 4/2 = 2
The variable x=2

Check required 2(2)+3=7 4+3 = 7 7=7

You have a lot of time to practice this so make it fun and do a lot of practice. This must become fluent in this grade. Students must also spend a fair amount of time reading a contextual situation, writing an equation that models the scenario, solves the scenario and then makes a complete statement about the solution.

**Blended Resources, Personal Learning Resources, Differentiated Learning Resources**

**CCSS Math Resources**

**Common Core stations 7th grade**

**Worksheets for teaching the inverse method (number tricks)**

**Google (quia solving 2 step equations for jeopardy, rags to riches, matching, concentration, or quizzes)**

**Post - Assessment Module 2 – Optional!**

**Probing questions:**

- What does it mean to be solution to an equation?
- How many solutions are there to these types of equations?
- What does order of operation have to do with solving linear equations?
- What does a variable mean in an equation?
- Identify 3 different ways to tell if two expressions are equal.
- Compare and contrast expressions and equations. Provide examples to support your response.
### Knowledge Packet for Grade 7 Module 3

**Strand(s):** EE  
**Topic A Use Properties of Operations to Generate Equivalent Expressions (7.EE.A.1, 7.EE.A.2)**  
**November 7 – November 14**

<table>
<thead>
<tr>
<th>Previous Grade Standard</th>
<th>Standard(s) for Grade/Course:</th>
<th>Next Grade Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.EE.A.3</td>
<td><strong>CCSS.MATH.CONTENT.7.NS.A.3</strong> Solve real-world and mathematical problems involving the four operations with rational numbers. <strong>CCSS.MATH.CONTENT.7.EE.B.4</strong> Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</td>
<td>8.EE.7A Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where $a$ and $b$ are different numbers).</td>
</tr>
<tr>
<td>6.EE.A.4</td>
<td><strong>Focus Standards:</strong> <strong>CCSS.MATH.CONTENT.7.EE.A.1</strong> Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.</td>
<td>8.EE.7B Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.</td>
</tr>
</tbody>
</table>

### Changes

**In 6th grade** students are fluent with one step equations, in 7th they become fluent at solving 2 step equations. Order of operation is extended from evaluating numerical values to using them to construct and deconstruct equations. Students move from a very concrete model of distributing and factoring (area model/tape diagrams) to a more fluent procedural method of distributing $a(b + c) = ab + c$.

**In 8th grade** students begin to solve more complex multi-step linear equations and must be able to determine the type of solution (zero, one or infinite). They also extend linear equations to systems of equations by substitution and elimination.
ac and begin to include rational coefficients.

**Big Ideas for Module 3 Topic A**

1. Students must be able to determine equivalent expressions (ie: Is the following equation true? \( \frac{1}{2}(x + 4) - 2 = \frac{1}{2}x + 2 \))

**Anchor Problem:**

**Preparing the Learner** - make sure you pretest the following concepts and address these before beginning this module and spend the time shoring these up before you begin this module.

- Evaluating using order of operation
- Simplifying expressions – combining like terms, distributing integer values
- Solving one step equations
- Translating expressions (math to English and English to math - more than, less than, sum, difference, is, was, per, in addition to, etc.)
- Inverse operations (How would you “undo” addition? What is the reciprocal of 10? What is the additive inverse of 5?)

**Recall**

1. I can deconstruct an equation using the order of operation. Ex: \( 2x + 3 = 7 \)

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Times 2</th>
<th>Plus 3</th>
<th>Equals 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
<td>4</td>
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</tbody>
</table>

2. I can determine whether a value is a solution to an equation by evaluating both sides for equality.
3. I can distribute a product to create a sum or difference and I can factor a sum or difference into a product of a rational number and a quantity

- Write an equivalent expression for \( 3(x+5)-2 \).
- Suzanne thinks the two expressions \( 2(3a-2)+4a \) and \( 10a-2 \) are equivalent? Is she correct? Explain why or why not?
- Write equivalent expressions for: \( 3a+12 \).
  - Possible solutions might include factoring as in \( 3(a+4) \), or other expressions such as \( a+2a+7+5 \).
- Jamie and Ted both get paid an equal hourly wage of $9 per hour. This week, Ted made an additional $27 dollars in overtime. Write an expression that represents the weekly wages of both if \( J = \) the number of hours that Jamie worked this week and \( T = \) the number of hours Ted worked this week? Can
4. I can determine if two equations are equivalent by evaluation, representation, and algebraic simplification.

<table>
<thead>
<tr>
<th>Making Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>1- I can write an expression that represents a contextual situation; or translate from words to mathematical notation and mathematical notation to words.</td>
</tr>
<tr>
<td>- Write the sum and then rewrite the expression in standard form by removing parentheses and collecting like terms.</td>
</tr>
<tr>
<td>a. 6 and ( p - 6 )</td>
</tr>
<tr>
<td>b. 10( w + 3 ) and ( -3 )</td>
</tr>
<tr>
<td>c. ( -x - 11 ) and the opposite of ( -11 )</td>
</tr>
<tr>
<td>d. The opposite of ( 4x ) and ( 3 + 4x )</td>
</tr>
<tr>
<td>e. ( 2g ) and the opposite of ( (1 - 2g) )</td>
</tr>
<tr>
<td>- Write the product and then rewrite the expression in standard form by removing parentheses and collecting like terms.</td>
</tr>
<tr>
<td>f. ( 7h - 1 ) and the multiplicative inverse of ( 7 )</td>
</tr>
<tr>
<td>g. The multiplicative inverse of ( -5 ) and ( 10v - 5 )</td>
</tr>
<tr>
<td>h. ( 9 - b ) and the multiplicative inverse of ( 9 )</td>
</tr>
<tr>
<td>i. The multiplicative inverse of ( \frac{1}{4} ) and ( 5t - \frac{1}{4} )</td>
</tr>
<tr>
<td>j. The multiplicative inverse of ( \frac{1}{10x} ) and ( \frac{1}{10x} - \frac{1}{10} )</td>
</tr>
</tbody>
</table>

**Teacher Ideas for Interaction**

**Eureka**

Topic A focuses on generating equivalent expressions. L1, L2, L3 provide good practice and discussion for combining like and delves into the concept of inverse as it relates to combining like terms (adding the opposite, multiplying by the reciprocal). Students are taught to both model and evaluate to prove expressions are equivalent. L4-5 focuses on distributing and factoring using the area model to generate equivalent expressions. L6 extends combining like terms and distributing with rational coefficients. Throughout this entire topic, have students show the expressions are equivalent 3 different ways: modeling, evaluating, and simplifying algebraically.

**Tasks**

7.EE.A.1 Illustrative Mathematics
- Writing Expressions (click here)
Activities
7.EE.A.1  NCTM Illuminations
• Distributing and Factoring Using Area (click here)

Representations:
• “Number tricks” – the inverse method
• Area Model
• Tape Diagrams
• Hands on equations
• 3D geometric models, 2D geometric models

Vocabulary
An Expression in Expanded Form, Expression in Standard, An Expression in Factored Form, Coefficient of the Term, Numerical Expression, Value of a Numerical Expression, Expression, Linear Expression, Equivalent Expressions, Identity, Term, Distribute, Factor, Properties of Operations (distributive, commutative, associative)

Probing Questions:
• Why does order matter when simplifying expressions?
• Which comes first multiply or divide?
• Why did we use the associative and commutative properties of addition?
• Can we use any order, any grouping when subtracting expressions? Explain.
• How does $3(x + y) = 3x + 3y$
• What is the opposite of a quantity?
• What is an inverse?
### Priority Standards:

**Strand(s):** EE, GB  
**Topic B:** Solve Problems Using Expressions, Equations, and Inequalities (7.EE.B.3, 7.EE.B.4, 7.G.B.5)  
**November 15 – November 30**

<table>
<thead>
<tr>
<th>Previous Grade Standard</th>
<th>Standard(s) for Grade/Course:</th>
<th>Next Grade Standard</th>
</tr>
</thead>
</table>
| **6.EE.B.6** | CCSS.MATH.CONTENT.7.EE.B.3  
Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation. | 8.EE.7b  
Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. |
| **6.EE.B.7** | **CCSS.MATH.CONTENT.7.EE.B.4.A**  
Solve word problems leading to equations of the form \( px + q = r \) and \( p(x + q) = r \), where \( p \), \( q \), and \( r \) are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? | 8.EE.8  
Analyze and solve pairs of simultaneous linear equations. |

**Focus Standard:**  
**CCSS.MATH.CONTENT.7.G.B.5**  
Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
In 6th grade students are fluent with one step equations, in 7th they become fluent at solving 2 step equations/inequalities. Equation solving then extends into multiple steps requiring distributing and combining like terms. Students also practice solving equations through geometry curricula involving triangle sum theorem, linear pairs, supplementary and complementary angles, and vertical angles. There is lots of geometry vocabulary that must be taught while students are practicing solving equations.

In 8th grade students develop fluency around solving multistep linear equations and then extend into solving systems of linear equations algebraically with substitution and elimination methods. Students extend the geometry content by solving for angles formed by parallel lines cut by a transversal.

**Big Ideas for Module 3 Topic B: (approximately 2 weeks)**

1. Students must FLUENTLY be able to solve 2 step equations and inequalities using the inverse method.
2. Students must be able to solve multistep equations involving combining like terms and distributing.
3. Students will use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

**Anchor Problem:**

**Recall/Skills**

1. I can FLUENTLY solve 2 step equations and inequalities
2. I can solve a multistep linear equations.

- Solve $\frac{5}{4}n + 5 = 20$
- Solve $\frac{1}{2}x + 3 > 2$ and graph your solution on a number line.

**Making Connections**

1- I can solve for a single variable in real world geometry formulas. Example: Given the volume, length and width of a rectangular prism, I can find the height.
2- I can solve real world problems by writing a linear equation and solving it.
3- I can find the measures of missing angles by solving an equation.

- A rectangle is twice as long as wide. One way to write an expression to find the perimeter would be $ww + ww + 2ww + 2ww$. Write the expression in two other ways.

  Solution: $6w$ OR $2(w) + 2(2w)$.

- An equilateral triangle has a perimeter of $6x + 15$. What is the length of each of the sides of the triangle? Solution: $3(2x + 5)$, therefore each side is $2x + 5$ units
| long.  
| • \( \angle A \) and \( \angle B \) are supplementary angles. \( m\angle A = 130^\circ, m\angle B = (x + 10)^\circ \). Find the value of \( x \).  
| • Amie had $26 dollars to spend on school supplies. After buying 10 pens, she had $14.30 left. How much did each pen cost?  
| • The sum of three consecutive even numbers is 48. What is the smallest of these numbers?  
| • Florencia has at most $60 to spend on clothes. She wants to buy a pair of jeans for $22 dollars and spend the rest on t-shirts. Each t-shirt costs $8. Write an inequality for the number of t-shirts she can purchase. |

### Teacher Ideas for Interaction

**Eureka**

*Topic B* is all about solving lots of equations and having students practice writing expressions and equations from context. **This is where students must develop fluency solving 2 step equations/inequalities.** Engage provides sprint exercises for this purpose in L12 and L15. L7-9 focus on solving problems in context by writing equations and solving them. L10-11 focus specifically on geometry problems where students are using equations to find measures of angles and sides. L12-15 extend the concept of solving equations (where there is a single solution (no solution and infinite solutions have not been discussed) to inequalities where there are infinite solutions. Take time developing the graphing portion where solutions are represented by dots on a number line. Model \( 2x = 6 \), \(|3|\), \( 2x < 6 \). Good activity provided in L15 for inequalities. Lots and lots of practice.

**Illustrative Mathematics**
- [Bookstore Account](#)  
- [Fishing Adventures 2](#)

**Inside Mathematics**
- [Growing Staircases](#)  
- [Once Upon A Time](#)

**MARS Shell Center**
- [Fencing](#)

**Youcubed.org**
- Number transformer challenge: [writing an equation from cubes/pictures](#)

**Representations:**
“Number tricks” – the inverse method
- Area Model
- Tape Diagrams
- Hands on equations
- 3D geometric models, 2D geometric models

**Vocabulary**
circle, diameter, circumference, pi, circular region or disc, Variable, Equation, Number Sentence, True or False Number Sentence, Truth Values of a Number Sentence, Inequality, Figure, Segment, Length of a Segment, Measure of an Angle, Adjacent Angles, Vertical Angles, Triangle, Square, Right Rectangular Prism, Cube, Surface of a Prism, consecutive

**Probing Questions:**
- What are some of the methods used to write products as sums?
- Why is the inequality reversed when you multiply or divide by a negative?
- What does it mean for an inequality to be preserved? What does it mean for the inequality to be reversed?
- Why is it possible to have more than one solution?
- What is the difference between consecutive integers and consecutive even or odd integers?
- What inequality symbol represents “is more than”? Why?
- What is a solution set of an inequality?
- How do you know when you need to use an inequality instead of an equation to model a given situation?
- Is it possible for an inequality to have exactly one solution? Exactly two solutions? Why or why not?
- When does a greater than become a less than?
- How are equations and expressions similar? How are they different?
**Knowledge Packet for Grade 7  Module 3**

**Priority Standards:**

Strand(s): EE, GB

**Topic C:** Use Equations and Inequalities to Solve Geometry Problems (7.G.B.4, 7.G.B.6)

December 1 – December 15

<table>
<thead>
<tr>
<th>Previous Grade Standard</th>
<th>Standard(s) for Grade/Course:</th>
<th>Next Grade Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.G.A.1</td>
<td><strong>CCSS.MATH.CONTENT.7.G.B.6</strong> Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.</td>
<td>The next time these topics are revisited is in High School Geometry</td>
</tr>
<tr>
<td></td>
<td><strong>Focus Standards:</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>CCSS.MATH.CONTENT.7.G.B.4</strong> Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.</td>
<td></td>
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<tr>
<td>6.G.A.2</td>
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<tr>
<td>6.G.A.4</td>
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</tbody>
</table>

Focus Standards:

**CCSS.MATH.CONTENT.7.G.B.4**

Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

**Focus Standards:**
<table>
<thead>
<tr>
<th>Changes</th>
<th>Changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>In 6\textsuperscript{th} grade students learn area is the number of</td>
<td></td>
</tr>
<tr>
<td>squares that covers and volume is the number of cubes that fills. They</td>
<td></td>
</tr>
<tr>
<td>find surface area of 3D objects by making nets. Volume is limited to</td>
<td></td>
</tr>
<tr>
<td>right prisms. In 7\textsuperscript{th}, students learn to define $\pi$</td>
<td></td>
</tr>
<tr>
<td>as circumference divided by diameter instead of a memorized 3.14.</td>
<td></td>
</tr>
<tr>
<td>Students must memorize the formulas for area of circle, triangle, and</td>
<td></td>
</tr>
<tr>
<td>parallelogram and the formula for circumference of a circle. The work</td>
<td></td>
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<tr>
<td>around area extends to composite areas and area/perimeter on the</td>
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<tr>
<td>coordinate plane, while the work around surface area continues with</td>
<td></td>
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<tr>
<td>nets. The work around volume continues to focus on right prisms but</td>
<td></td>
</tr>
<tr>
<td>extends to composite solids.</td>
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</tbody>
</table>

**Anchor Problem:**


## Knowledge Packet for Grade 7 Module 3

### Recall/Skills - Level I

1. I can find the area of any polygon by dividing it into rectangular or triangular pieces and finding the sum of the areas.

2. Find the area of the trapezoid shown below using the formulas for rectangles and triangles.

### Making Connections

1. I can define pi as the quotient of the circumference and the diameter.

   (Students measure the circumference and diameter of several circular objects in the room (clock, trash can, door knob, wheel, etc.). Students organize their information and discover the relationship between circumference and diameter by noticing the pattern in the ratio of the measures. Students write an expression that could be used to find the circumference of a circle with any diameter and check their expression on other circles.)

2. I can give an informal derivation of the relationship between the circumference and area of a circle.

   (Students will use a circle as a model to make several equal parts as you would in a pie model. The greater the number of cuts, the better. The pie pieces are laid out to form a shape similar to a parallelogram. Students will then write an expression for the area of the parallelogram related to the radius (note: the length of the base of the parallelogram is half the circumference, or πr, and the height is r, resulting in an area of πr². Extension: If students are given the circumference of a circle, could they write a formula to determine the circle’s area or given the area of a circle, could they write the formula for the circumference?)

- The seventh grade class is building a mini golf game for the school carnival. The end of the putting green will be a circle. If the circle is 10 feet in diameter, how many square feet of grass carpet will they need to buy to cover the circle? How might you communicate this information to the salesperson to make sure you receive a piece of carpet that is the correct size?

- Choose one of the figures shown below and write a step by step procedure for determining the area. Find another person that chose the same figure as you did. How are your procedures the same and different? Do they yield the same result?

- A cereal box is a rectangular prism. What is the volume of the cereal box? What is the surface area of the cereal box? (Hint: Create a net of the cereal box and use the net to calculate the surface area.) Make a poster explaining your work to share with the class.

- Find the area of a triangle with a base length of three units and a height of four units.
### Preparing the Learner (approximately 3 weeks)

Make sure you pre-assess students to determine if they understand basic concepts of area, perimeter, surface area, and volume.

### Teacher Ideas for Interaction

#### Eureka

Lessons 16-18 focus heavily on moving students from $\pi = 3.14$ to $\frac{c}{d}$. Students should do the pi lab where they measure the circumference and diameter of multiple real life circular objects and start to compare the results of $\frac{c}{d}$. Discussion should include inaccuracy of measurement but students should get the idea. They also focus heavily on applications of circumference and area in context (including quarter circles and half circles) and on area of composites involving circles. The focus is still on solving equations so they are provided parts of the geometric formulas and must solve for a single variable. Make sure students are memorizing basic area and circumference formulas. Lesson 19 moves students to the coordinate plane to derive area formulas for parallelogram and triangles and students begin to understand height must be perpendicular to the base while lesson 20 provides many composite area problems to practice with. Lessons 21-22 focus on surface area of solids via making nets and finding the sum of the areas. Lessons 23-24 focus on volume of prisms by finding the area of the base and multiplying by the height while lessons 25-26 involve solving real problems involving both surface area and volume. Students will find both fractional parts of solids as well as the remaining space when a solid is inside a solid. Problems can range from very low to very high.

### Activities

- **MARS Shell Center**
  - Using Dimensions: [Designing a Sports Bag](click here)

- **NCTM Illuminations**
  - [Popcorn, Anyone?](click here)

**Illustrative Mathematics**
• Designs (click here)  
• Eight Circles (click here)  
• Stained Glass (click here)  

Inside Mathematics  
• Circular Reasoning (click here)  

MARS Shell Center  
• Historic Bicycle (click here)  
• Sports Bag (click here)  

Illustrative Mathematics  
• Sand Under the Swing Set (click)  

MARS Shell Center  
• Fearless Frames (click)  

• instructional video for circumference: (click here)  
• Khan academy – area of a circle (click here)  
• Geogebra is a great resource for circumference of a circle  
• Area and perimeter (click here)  

**Representations:**  
• 3D models (preferably that break down into nets)  
• 2D representations of 3D models (nets)  
• Grid paper – coordinate plane  
• Structure of inverse method of solving linear equations  

**Probing Questions:**  
• In the pi lab, why didn't you get exactly 3.1415... for your constant of proportionality?  
• Is it possible to get a number that is greater than 3.5 for pi (if you measured accurately)? Explain.  
• How are the diameter and the radius related?  
• Students often say that area is the 'inside' of a circle and circumference is the 'outside.' Why isn't this accurate? Explain your thinking?  
• Give real life examples of perimeter or circumference?
- Why isn't circumference units squared?
- How is the area of a circle like the area of a rectangle?
- If you have the diameter of a circle how can you find the area?
- How is using grid paper to find area easier or harder?
- In what situations is knowing surface area useful?
- What was your thought process as you took the net and attempted to draw a sketch of the shape without knowing what the actual object is?
- How is surface area like covering and volume like filling?
- How is area related to surface area and nets?
- When is knowing volume helpful?
- What strategy do you use to determine the face that is the Base of a 3D object?
- When is it better to think about Volume as Bh instead of lwh?
## Knowledge Packet for Grade 7 Module 4: revised 8/10/16

**Strand(s):** RP

**Topic A:** Finding the Whole (7.RP.A.1, 7.RP.A.2c, 7.RP.A.3) January 4 - January 11 (includes both A and B as anchor problem) (6 days)

<table>
<thead>
<tr>
<th>Previous Grade Standard</th>
<th>Standard(s) for Grade/Course:</th>
<th>Next Grade Standard</th>
</tr>
</thead>
</table>
| CCSS.MATH.CONTENT.6.RP.A.3b | **This is tested in Module 1 and Module 4 Topic C/D**  
Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*  
CCSS.Math.Content.7.RP.A.2 | CCSS.MATH.CONTENT.8.EE.B.5 | **Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.**  
For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.  
CCSS.MATH.CONTENT.8.EE.B.6 | **Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.** |
| CCSS.MATH.CONTENT.6.RP.A.3c | **Focus Standards:**  
Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity mean 30/100 time the quantity); solve problems involving finding the whole, given a part and the percent.  
CCSS.MATH.CONTENT.7.RP.A.1 |  |  |
|  | Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.  
CCSS.MATH.CONTENT.7.RP.A.3- Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. |  |  |

**Changes**

Solving problems involving percents is a 6th grade standard but many 7th graders still struggle with the concept. Module 3’s focus on algebra prepares students to move from the visual models used for percents in Grade 6 to algebraic equations in Grade 7. This needs to be a fluency piece in 7th.

**Anchor Problem:** click for sample Anchor problem

**Changes**

Students in 8th extend their understanding of proportional relationships with equations and graphs with different content. The focus moves to an analysis phase of equations and graphs including but not limited to rate of change and transformations. Students also move toward building equations that represent proportional relationships.
As you create your inventory how did you decide what percent of each type of items you were going to keep in your inventory?

How would you decide what 15% of your inventory is? How would you determine what the percent of your inventory 30 is?

In our business we are noticing that the inventory is not selling very well what is the part and whole for your inventory compared to what you started out at?

What is the difference in finding the amount the item cost before the sales versus finding the sales price of the same item?

Explain what your business percent of increase of sales, or percent of decrease of sales?

Final: Students will peer edit and share their end store inventory.

Preparing the Learner: students must be able to translate between decimals, percents, and fractions so keep doing the sprints until we are fluent! Make sure students understand the major concept of percent.

Big ideas for this module:

- translate between fractions, decimals, and percents
- fluency with percents and solving percent problems

Recall/Skills:

- I can convert between fractions, decimals, and percents.

- I can order rational numbers on the number line

- .333333333333 is what fraction?

- Put the following fractions in order of size, starting with the one with the least value:

  \[
  \begin{array}{cccccc}
  \frac{3}{4} & \frac{9}{16} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8}
  \end{array}
  \]

  Least, Greatest

  Explain your method for doing this.
Making Connections

- I can solve real life problems involving percents.
- After eating at a restaurant, your bill before tax is $52.60. The sales tax rate is 8%. You decide to leave a 20% tip for the waiter based on the pre-tax amount. How much is the tip you leave for the waiter? How much will the total bill be, including tax and tip?
- Gas prices are projected to increase 124% by April 2015. A gallon of gas currently costs $4.17. What is the projected cost of a gallon of gas for April 2015?
Teacher Ideas for Interaction

Eureka

In Module 4, students deepen their understanding of ratios and proportional relationships from Module 1 (7.RP.A.1, 7.RP.A.2, 7.RP.A.3, 7.EE.B.4, 7.G.A.1) by solving a variety of percent problems. They convert between fractions, decimals, and percents to further develop a conceptual understanding of percent and use algebraic expressions and equations to solve multi-step percent problems (7.EE.B.3). L1 is invaluable as a resource to move students from a concrete understanding to conceptual fluency. Making sure students are fluent with this skill will make or break success later in the module. The opening activity is good, and there are multiple activities (matching, I have who has, modeling, sprints) to practice within this lesson. You could create multiple stations within this lesson alone. There are more sprints in L3 and L6. These are important to repeat these several times to help students move toward fluency. An initial focus on relating 100% to “the whole” serves as a foundation for students. Topic A builds on students’ conceptual understanding of percent from Grade 6 (6.RP.3c), and relates 100% to “the whole.” Students represent percents as decimals and fractions and extend their understanding from Grade 6 to include percents greater than 100%, such as 225%, and percents less than 1%, such as $\frac{1}{2}$% or 0.5%. They understand that, for instance, 225% means $\frac{225}{100}$, or equivalently, $\frac{2.25}{1} = 2.25$ (7.RP.A.1). Students use complex fractions to represent non-whole number percents (e.g., $12\frac{1}{2}$% = $\frac{12\frac{1}{2}}{100} = \frac{1}{8} = 0.125$).

L2-L3 provide opportunities to practice all types of percent problems described above. L4 focuses specifically on increase decrease problems using tape diagrams and double number lines (modeling). They write equations to solve multi-step percent problems and relate their conceptual understanding to the representation: Quantity = Percent × Whole (7.RP.A.2c). Students solve percent increase and decrease problems with and without equations (7.RP.A.3). For instance, given a multi-step word problem where there is an increase of 20% and “the whole” equals $200, students recognize that $200 can be multiplied by 120%, or 1.2, to get an answer of $240. They use visual models, such as a double number line diagram, to justify their answers. In this case, 100% aligns to $200 in the diagram and intervals of fifths are used (since 20% = $\frac{1}{5}$) to partition both number line segments to create a scale indicating that 120% aligns to $240.

L5-L6 should be consider additional resources as they get into mental strategies with percents and lots of practice solving problems with percents.

L1 has a great opening activity to find out what your students understand about percents. There is abundant practice included here on renaming numbers as decimals, fractions and percents. There is also a great “I have who has” class builder as well as a print for renaming decimals, fractions, and percents which could be used several times.

L2,L3,L4 &L5 provide good modeling using tape diagrams and double number lines and good conceptual development. At the end of L3 are additional sprints to help students build fluency around percents. A lot of practice included as well.
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L6 is designed for fluency only – lots of practice and also contains more sprints to use with students. The sprints should be used multiple times so students work to improve their own score.

Blended Resources, Personal Learning Resources, Differentiated Learning Resources

CCSS Math Resources

Common Core stations 7th grade

Quia (google “quia percent problems” for jeopardy, rags to riches, matching, concentration, or quizzes)

Mars Shell Center
- 25% Sale
- Ice Cream
- Sale!

Howard County
- Comparison shopping
- Movie Revenue
- 7th grade dance

Khan Academy-proportional relationships
Math playground – thinking blocks

Pre-Assessment module 4

Representations:
- Double number line
- Expressions
- Area Model
- Tape Diagram
- Equations and Inequalities
- Number Line
- Coordinate Plane
- Geometric Figures
- Protractor
## Nets for Three-Dimensional Figures

### Vocabulary:

- **An Expression in Expanded Form** *(description)* An expression that is written as sums (and/or differences) of products whose factors are numbers, variables, or variables raised to whole number powers is said to be in expanded form. A single number, variable, or a single product of numbers and/or variables is also considered to be in expanded form.

- **An Expression in Standard Form** *(description)* An expression that is in expanded form where all like-terms have been collected is said to be in standard form.

- **An Expression in Factored Form** *(middle school description)* An expression that is a product of two or more expressions is said to be in factored form.

- **Coefficient of the Term** *(description)* The number found by multiplying just the numbers in a term together is called the coefficient of the term.

- **Circle** *(description)* Given a point $C$ in the plane and a number $r > 0$, the circle with center $C$ and radius $r$ is the set of all points in the plane that are distance $r$ from the point $C$.

- **Diameter of a Circle** *(description)* The diameter of a circle is the length of any segment that passes through the center of a circle whose endpoints lie on the circle. If $r$ is the radius of a circle, then the diameter is $2r$.

- **Circumference** *(description)* The length around a circle.

- **Pi** *(description)* The number $\pi$, denoted $\pi$, is the value of the ratio given by the circumference to the diameter, that is, $\pi = \text{(circumference)}/(\text{diameter})$.

- **Circular Region or Disk** *(description)* Given a point $C$ in the plane and a number $r > 0$, the circular region (or disk) with center $C$ and radius $r$ is the set of all points in the plane whose distance from the point $C$ is less than or equal to $r$. The interior of a circle with center $C$ and radius $r$ is the set of all points in the plane whose distance from the point $C$ is less than $r$.

### Familiar Terms and Symbols

- **Variable** *(middle school description)* (letter that represents a numerical value)

- **Numerical Expression** *(middle school description)* 2 or more numbers combined with 1 or more operations

- **Value of a Numerical Expression** *(description)* when all variables are replaced with numerical values and a single value is produced

- **Expression** *(middle school description)* combination of numbers and/or letters without an equal sign joined by at least one operation

- **Linear Expression** – at least one variable and/or number combined with at least one operation without an equal sign

- **Equivalent Expressions** – when all variables are replaced with values, the value of the expression on the left is the same value as you get for the expression to the right of the equal sign

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1 “Distance around a circular arc” is taken as an undefined term in G-CO.1.
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- **Equation** – 2 expressions including an equal sign
- **Number Sentence** – an equation strictly with numbers
- **True or False Number Sentence** - if when all variables are replaced by numerical values and both expressions are equal it is a true sentence; if however the values are not equal, it is a false number sentence
- **Truth Values of a Number Sentence** – those values which make the value of both expressions on either side of the equal sign equal
- **Identity** – addition it is 0; 3+0=3 – doesn’t change the identity, multiplication it is 1: 3×1 = 3
- **Term** - a single letter or number or combination of numbers and letters without operations
- **Distribute** - the multiplication of two numbers or expressions which produces a sum or difference
- **Factor** – one of the two numbers that is multiplied together to get a different number
- **Properties of Operations** (distributive, commutative, associative)
- **Inequality** – not equal
- **Figure** – a combination of points, lines, and planes
- **Segment** – part of a line with 2 endpoints
- **Length of a Segment** – the absolute value the distance between two points
- **Measure of an Angle** – the number of degrees an angle opens up on the unit circle.
- **Adjacent Angles** – 2 angles who share a side
- **Vertical Angles** – 2 angles created by intersecting lines
- **Triangle** – a polygon with three sides
- **Square** – a polygon with 4 equal sides, adjacent sides are perpendicular
- **Right Rectangular Prism** – a 3 dimensional figure with 6 faces
- **Cube** – a 3 dimensional figure with 6 equal faces.
- **Surface of a Prism** – the faces of the prism – (surface area – number of squares it takes to cover the surface)
**Priority Standard:**

Strand(s): Topic B: Percent Problems Including More than One Whole (7.RP.A.1, 7.RP.A.2, 7.RP.A.3, 7.EE.B.3)  
January 12 – January 19 (includes both A and B as anchor problem) (5 days)

<table>
<thead>
<tr>
<th>Previous Grade Standard</th>
<th>Standard(s) for Grade/Course:</th>
<th>Next Grade Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCSS.Math.Content.6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</td>
<td>CCSS.MATH.CONTENT.7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.</td>
<td>CCSS.MATH.CONTENT.8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading).</td>
</tr>
<tr>
<td>CCSS.Math.Content.6.EE.A.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.</td>
<td></td>
<td>CCSS.Math.Content.8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct point on a non-vertical line in the coordinate plane; derive the equation y=mx for a line through the origin and the equation y=mx + b for a line intercepting the vertical axis at b.</td>
</tr>
<tr>
<td>CCSS.Math.Content.6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, &quot;The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.&quot; &quot;For every vote candidate A received, candidate C received nearly three votes.&quot;</td>
<td>CCSS.MATH.CONTENT.7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.</td>
<td>CCSS.MATH.CONTENT.8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.</td>
</tr>
<tr>
<td></td>
<td>CCSS.MATH.CONTENT.7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax,</td>
<td>CCSS.MATH.CONTENT.8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs</td>
</tr>
</tbody>
</table>
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- markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
- consisting of an input and the corresponding output.

<table>
<thead>
<tr>
<th>Changes</th>
<th>Changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>From solving ratio and rate real-world problems to representing proportional relationships.</td>
<td>From percent representations to scientific notation.</td>
</tr>
<tr>
<td>From writing and solving equations with nonnegative rational numbers to solving multi-step mathematical problems with positive and negative rational numbers in any form.</td>
<td>From identifying the constant of proportionality to using the unit rate as the slope in a graph.</td>
</tr>
<tr>
<td>From describing a ratio relationship between two quantities to computing unit rate associated with ratios.</td>
<td>From using equations and tables to using functions and ordered pairs.</td>
</tr>
<tr>
<td>New: unit rate, proportional relationships</td>
<td>New: Scientific notation and deriving a function using slope.</td>
</tr>
</tbody>
</table>

**Anchor problem:** Have students discuss their favorite athletes. What do they like about the athlete, what do they enjoy watching them play? Discuss what math goes into the sport... What math does the coach use to decide whether or not the athlete should play in the game? On the team’s roster what data/information would you want access to about the players/athletes to make amount of play time decisions? Have students pick two athletes from the same sport. Students will pull data from two different seasons (for example, shots made vs. shots taken, goals made vs. goals attempted) for both athletes. Students will create a data table and insert their athletes’ data. This data will start in fraction form and then find the percentages for each season and each athlete. Who has the better average for each season? What is the average of each athlete when you combine seasons? Who has the better average now?

A baseball player’s batting average is the fraction of times at bat when the player gets a hit. Below is a table showing the number of hits and the number of times at bat for two Major League Baseball players during two different seasons:

<table>
<thead>
<tr>
<th>Season</th>
<th>Derek Jeter</th>
<th>David Justice</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>12 hits, 48 times at bat</td>
<td>104 hits, 411 times at bat</td>
</tr>
<tr>
<td>1996</td>
<td>183 hits, 582 times at bat</td>
<td>45 hits, 140 times at bat</td>
</tr>
</tbody>
</table>
1. For each season, find the players’ batting averages. Who has the better batting average?
2. For the combined 1995 and 1996 seasons, find the players’ batting averages. Who has the better batting average?
3. Are the answers to (a) and (b) consistent? Explain.

**Big ideas for this module:**

**Solving real life problems involving percents (gratuity, discounts, simple interest, tax, etc.)**

<table>
<thead>
<tr>
<th>Recall/Skills</th>
<th>Making Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>- I can solve percent problems for part, percent, and whole</td>
<td>- A $300 mountain bike is discounted by 30% and then discounted an additional 10% for shoppers who arrive before 5:00 a.m. Find the sales price of the bicycle.</td>
</tr>
<tr>
<td>- I can express a 5% increase as 105% and a 10% decrease as 90%</td>
<td>- Kacey and her three friends went out for lunch, and they wanted to leave a 15% tip. The receipt shown below lists the lunch total before tax and tip. The tip is on the cost of the food plus tax. The sales tax rate in Pleasantville is 3.5%.</td>
</tr>
</tbody>
</table>
| - I can calculate simple interest using I=PRT | - A store that sells skis buys them from a manufacturer at a wholesale price of $57. The store’s markup rate is 50%.
  a. What price does the store charge its customers for the skis? |
b. What percent of the original price is the final price? Show your work.
c. What is the percent increase from the original price to the final price?

- Terrence and Lee were selling magazines for a charity. In the first week, Terrance sold 30% more than Lee. In the second week, Terrance sold 8 magazines, but Lee did not sell any. If Terrance sold 50% more than Lee by the end of the second week, how many magazines did Lee sell?

### Teacher Ideas for Interaction

**Eureka**

All of the lessons in topic B are intended to be practical application scenarios of markups and markdowns, simple interest, tax situations etc. Choosing a few problems to develop at a deep level of understanding would be better than exposing students to every lesson quickly.

In Topic B, students create algebraic representations and apply their understanding of percent from Topic A to interpret and solve multi-step word problems related to markups or markdowns, simple interest, sales tax, commissions, fees, and percent error (7.RP.A.3, 7.EE.B.3). They apply their understanding of proportional relationships from Module 1, creating an equation, graph, or table to model a tax or commission rate that is represented as a percent (7.RP.A.1, 7.RP.A.2). Students solve problems related to changing percents and use their understanding of percent and proportional relationships to solve the following: A soccer league has 300 players, 60% of whom are boys. If some of the boys switch to baseball, leaving only 52% of the soccer players as boys, how many players remain in the soccer league?

Students determine that, initially, 100% − 60% = 40% of the players are girls and 40% of 300 equals 120. Then, after some boys switched to baseball, 100% − 52% = 48% of the soccer players are girls, so 0.48p = 120, or \( p = \frac{120}{0.48} \). Therefore, there are now 250 players in the soccer league.

In Topic B, students also apply their understanding of absolute value from Module 2 (7.NS.A.1b) when solving percent error problems. To determine the percent error for an estimated concert attendance of 5,000 people, when actually 6,372 people attended, students calculate the percent error as:

\[
\frac{|5000 - 6372|}{|6372|} \cdot 100\% = 21.5\%.
\]

The focus on the remaining topics including topic B is to deeply engage students into real life problem solving. It is highly recommended that you choose a few problems from this topic and develop them deeply as appose to skimming through all of the lessons. **Lesson 9** has some really good modeling using tape diagrams and **lesson 10** contains another sprint developing fluency around percents for all students. L7 focuses on solving mark up and mark down problems. L8 focuses on margin of error problems and should be considered an extension lesson since it has a loose connection to the standard. L9 concentrates on problems involving a change in the percent. L10 entails computing simple interest on a savings account while L11 presents calculating tax and commissions.
MARS Shell Center
• Increasing and Decreasing Quantities by a Percent

Probing questions:
• Why would a store owner mark-up or mark-down an item?
• What does supply and demand have to do with marking up or marking down an item?
• Why will you need to calculate interest in the real-world?
• If a peer borrows money from you, how would you calculate how much they owe you if you charge them interest?
• What is gratuity and what services or goods do we usually leave gratuity for?
• What types of items does a salesperson earn commission for?
• What does tax-exempt mean and why would a school be tax-exempt?

Anchor questions:
• What is the salary for your athlete? Did the salary increase or decrease from their previous contract? What was the mark-up or mark-down rate?
• Research how many jersey’s or other items (shoes) your athlete sold this past year. Find out the amount of commission your athlete makes for every item sold and what their total amount was made this past year.
• When looking at the salary of your athlete, defend if their salary is appropriate. Support your claim with their performance data, merchandise sold, and any other pieces of evidence you have collected.
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<tbody>
<tr>
<td>CCSS.MATH.CONTENT.6.RP.A.3</td>
<td>CCSS.Math.Content.7.RP.A.2 Recognize and represent proportional relationships between quantities. d) Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.</td>
<td>CCSS.Math.Content.8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</td>
</tr>
<tr>
<td>Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</td>
<td>CCSS.MATH.CONTENT.7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.</td>
<td>CCSS.Math.Content.8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</td>
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<tr>
<td>CCSS.MATH.CONTENT.6.RP.A.3.B</td>
<td>Solve unit rate problems including those involving unit pricing and constant speed.</td>
<td>From whole numbers or nonnegative rational numbers to all rational numbers. From single step ratio problems to multistep. From the concept of using ratios and rates to solve real world problems to identifying the constant of proportionality. From using tape diagrams, double number lines or equations to using tables, graphs, equations, diagrams and verbal descriptions of proportional relationships (unit rate). From using the term &quot;constant of proportionality&quot; to using the term &quot;slope&quot;. From finding constant of proportionality from a table, graph or equation to finding rate of change/slope. From proportional relationships to linear relationships. From writing proportional equations to writing linear equations. From solving px=q to p(x + q) = r. From using equations and tables to using functions and ordered pairs. From creating scale drawings to deriving slope from the drawing.</td>
</tr>
</tbody>
</table>
**Anchor Problem:**
Have the students create or find a logo related to an athlete they investigated in the previous topic. Then have them scale the logo by either 75% or 125%. Next have students rescale the logo to an enlargement or reduction of their choice and have other students find their scale factor as a percent.

**Preparing the learner:** Again make sure you are giving the pretest several days before you start the unit. Specifically students need to identify figures as enlargements or reductions and they need to know how to fluently change percents to decimals or fractions.

**Big ideas for this module:**

Revisiting scale drawings (reductions and enlargements) using percents

**Recall/Skills:**
- I can find a given percentage of a given number. (ie: Find 70% of 12 cm)
- I can represent a ratio as a percent reduction or enlargement.
- I can represent a 20% enlargement as 120% of the original and a 20% reduction as 80% of the original.

- 306 is what percent of 900?
- What is 39% of 400?
- A regular octagon is an eight-sided polygon with side lengths that are all equal. All three octagons are scale drawings of each other. Use the side lengths to compute each scale factor as a percent.
- The scale factor from Drawing 1 to Drawing 2 is 60%. Find the scale factor from Drawing 2 to Drawing 1. Explain your reasoning.

**Making Connections**
- I can enlarge or reduce a scale drawing based on a percent. (ie: 20% reduction, 50% enlargement)
- Create a scale drawing of the picture below using a scale factor of 60%. Write three equations that show how you determined the lengths of three different parts of the resulting picture.
Teacher Ideas for Interaction

Eureka

Students revisit scale drawings in Topic C to solve problems in which the scale factor is represented by a percent (7.RP.A.2b, 7.G.A.1). They understand from their work in Module 1, for example, that if they have two drawings where if Drawing 2 is a scale model of Drawing 1 under a scale factor of 80%, then Drawing 1 is also a scale model of Drawing 2, and that scale factor is determined using inverse operations. Since \(80\% = \frac{4}{5}\), the scale factor is found by taking the complex fraction \(\frac{1}{\frac{4}{5}}\) or \(\frac{5}{4}\), and multiplying it by 100%, resulting in a scale factor of 125%. As in Module 1, students construct scale drawings, finding scale lengths and areas given the actual quantities and the scale factor (and vice-versa); however, in this module the scale factor is represented as a percent. Students are encouraged to develop multiple methods for making scale drawings. Students may find the multiplicative relationship between figures; they may also find a multiplicative relationship among lengths within the same figure.

Lessons 12, 13 and 15 have great Opening Exercises that can directly be used at Explores. In Lesson 12, students extend their understanding of scale factor to include scale factors represented as percents. Students know the scale factor to be the constant of proportionality, and they create scale drawings when given horizontal and vertical scale factors in the form of percents (7.G.A.1, 7.RP.A.2b). In Lesson 13, students recognize that if Drawing B is a scale drawing of Drawing A, then one could also view Drawing A as being a scale drawing of Drawing B; they compute the scale factor from Drawing B to Drawing A and express it as a percentage. Also in this lesson, students are presented with three similar drawings—an original drawing, a reduction, and an enlargement—and, given the scale factor for the reduction (as a percentage of the original) and the scale factor for the enlargement (as a percentage of the original), students compute the scale factor between the reduced image and the enlarged image, and vice versa, expressing each scale factor as a percentage. In Lesson 14, students compute the actual dimensions when given a scale drawing and the scale factor as a percent. To solve area problems related to scale drawings, in Lesson 15, students use the fact that an area, \(AA'\), of a scale drawing is \(k^2\) times the corresponding area, \(AA\), in the original picture (where \(kk\) is the scale factor). For instance, given a scale factor of 25%, students convert to its fractional representation of 1/4 and know that the area of the scale drawing is \((\frac{1}{4})^2\) or \(\frac{1}{16}\) the area of the original picture and use that fact to problem solve.
Probing questions:

- How do you determine whether it is an enlargement or reduction from figure 1 to figure 2?
- Describe the process for using the side lengths to determine the scale factor as a percent.
- What is the significance of the scale factor as it relates to 100%?
- How are scale factor, unit rate, and constant of proportionality related?
- When a scale factor is given as a percent, why is it best to convert the percent to a fraction?
- If you reverse the order and compare Drawing 2 to Drawing 1, it appears Drawing 1 is smaller; therefore, it is a reduction. What do you know about the scale factor of a reduction?
- Referring to the octagons in the skill section above, why are all three octagons scale drawings of each other?
- Which corresponding parts did you choose to compare when calculating the scale factor, and why did you choose them?
- How do you compute the scale factor when given a figure and a scale drawing of that figure?
- How do you use the scale factor to compute the lengths of segments in the scale drawing and the original figure?
- Is it necessary to find the area of each drawing to determine the ratio of areas of the scale drawing to the original drawing, if the scale factor is known?
- Why is the scale factor often given as a percent or asked for as a percent but the area relationship is calculated as a fraction? Why can’t a percent be used for this calculation?
- If you know a length in a scale drawing and its corresponding length in the original drawing, how can you determine the relationship between the areas of the drawings?
Knowledge Packet for Grade 7 Module 4: revised 8/10/16

Strand(s):

<table>
<thead>
<tr>
<th>Previous Grade Standard</th>
<th>Standard(s) for Grade/Course:</th>
<th>Next Grade Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCSS.MATH.CONTENT.6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</td>
<td><strong>Focus Standards:</strong> (These were already assessed.) <strong>CCSS.Math.Content.7.RP.A.2</strong> Recognize and represent proportional relationships between quantities. b) Represent proportional relationships by equations. For example, if total cost $t$ is proportional to the number $n$ of items purchased at a constant price $p$, the relationship between the total cost and the number of items can be expressed as $t = pn$.</td>
<td>CCSS.Math.Content.8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT.6.RP.A.3.A Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. CCSS.MATH.CONTENT.6.RP.A.3.B Solve unit rate problems including those involving unit pricing and constant speed. CCSS.MATH.CONTENT.6.RP.A.3.C Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. CCSS.MATH.CONTENT.6.RP.A.3.D Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. 6.EE.B.6</td>
<td><strong>CCSS.MATH.CONTENT.7.EE.B.3</strong> Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.</td>
<td>CCSS.Math.Content.8.EE.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two $(x, y)$ values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</td>
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<td></td>
<td><strong>CCSS.MATH.CONTENT.7.RP.A.3</strong> Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</td>
<td>CCSS.Math.Content.8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and</td>
</tr>
</tbody>
</table>
Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

6.EE.B.7
Solve real-world and mathematical problems by writing and solving equations of the form \( x + p = q \) and \( px = q \) for cases in which \( p, q \) and \( x \) are all nonnegative rational numbers.

Choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

8.EE.7b Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8.EE.8 Analyze and solve pairs of simultaneous linear equations.

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<tr>
<th>Changes</th>
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<tr>
<td>From single step ratio problems to multistep. Solving now includes multi-step equations and inequalities with rational numbers. Equations in 7th grade are solved in context with an emphasis on rational numbers and multi-step processes. Heavily used in the geometry context in 7th grade.</td>
<td>From using the term &quot;constant of proportionality&quot; to using the term &quot;slope&quot;. From finding constant of proportionality from a table, graph or equation to finding rate of change/slope. From proportional relationships to linear relationships. From writing proportional equations to writing linear equations. 7th grade does not use scientific notation at all, which is taught in 8th grade. When equations are graphed in 7th grade they are always proportional and this is extended to all linear equations in 8th. 7th grade solves one equation at a time where as 8th grade extends to basic systems of linear equations.</td>
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</tbody>
</table>

**Anchor Problem:**
Preparing the learner: This topic should be considered an extension and should be used to differentiate for top end students. This is an optional topic that continues to ask students to problem solve with percents using content that are both real life and connected to other mathematical content. If you have the time, students need to practice these types of percent problems. Be sure to make sure they can model as well.

Big ideas for this module:

Real life problems involving multiple percents.

Recall/Skills:
- I can find a given percentage of a given number. (ie: Find 70% of 12 cm)
- A school has 60% girls and 40% boys. If 20% of the girls wear glasses and 40% of the boys wear glasses, what percent of all students wears glasses? Model with a tape diagram.

Making Connections
- I can use percents to solve population problems.
- I can use percents to solve mixture problems.
- I can use percents to solve counting problems.
- Jodie spent 25% less buying her English reading book than Claudia. Gianna spent 9% less than Claudia. Gianna spent more than Jodie by what percent?
- A 25% vinegar solution is combined with triple the amount of a 45% vinegar solution and a 5% vinegar solution resulting in 20 milliliters of a 30% vinegar solution. Determine an equation that models this situation, and explain what each part represents in the situation. Solve the equation and find the amount of each of the solutions that were combined.

Teacher Ideas for Interaction

Eureka
The problem-solving material in Topic D provides students with further applications of percent and exposure to problems involving population, mixtures, and counting, in preparation for later topics in middle school and high school mathematics and science. L16 uses heavy modeling with tape diagrams as they practice percents with populations. Students apply their understanding of percent (7.RP.A.2c, 7.RP.A.3, 7.EE.B.3) to solve word problems in which they determine, for instance, when given two different sets of 3-letter passwords and the percent of 3-letter passwords that meet a certain criteria, which set is the correct set. Or, given a 5-gallon mixture that is 20% pure juice, students determine how many gallons of pure juice must be added to create a 12-gallon mixture that is 40% pure juice by writing and solving the equation \(0.2(5) + j = 0.4(12)\), where \(j\) is the amount of pure juice added to the original mixture. They also see percent applied to other areas of math and science. In Lessons 16 and 17, students represent and solve population and mixture problems using algebraic expressions and equations, along with their foundational understanding from Topic A of the equation Quantity = Percent \times Whole (7.RP.A.2c). Topic D concludes with Lesson 18, where students solve counting problems involving percents, preparing them for future work with probability.

Howard County - click to view various resources

Post Assessment Module 4

Common Assessment Module 4

Representations:
- Tables
- Equations
- Expressions
- Double number lines
- Tape Diagrams

Probing questions:
- What is the importance of defining the variable for percent population problems?
- How do tape diagrams help to solve for percent population problems?
- How does using a table help you solve mixture problems?
- What is the general structure of the expressions for mixture problems?
- How do mixture and population problems compare?
- What information must be known to find the percent of possible outcomes for a counting problem?
## Knowledge Packet for Grade 7 Module 5: Revised 8/10/16

### Priority Standard:

<table>
<thead>
<tr>
<th>Strand(s): SP</th>
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<tbody>
<tr>
<td>Topic A: Calculating and Interpreting Probabilities (7.SP.C.5, 7.SP.C.6, 7.SP.C.7, 7.SP.C.8a, 7.SP.C.8b)</td>
</tr>
<tr>
<td>February 2 - February 13</td>
</tr>
<tr>
<td>Topic B: Estimating Probabilities (7.SP.C.6, 7.SP.C.7, 7.SP.C.8c)</td>
</tr>
<tr>
<td>February 14 – February 22</td>
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</tbody>
</table>

<table>
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<tr>
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<tbody>
<tr>
<td>CCSS.MATH.CONTENT.5.NF.A.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)</td>
<td>CCSS.MATH.CONTENT.7.SP.C.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.</td>
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<tr>
<td><strong>CCSS.MATH.CONTENT.6.NS.A.1</strong> Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.</td>
<td>a) Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.</td>
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<tr>
<td>CCSS.MATH.CONTENT.6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems</td>
<td>b) Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., &quot;rolling double sixes&quot;), identify the outcomes in the sample space which compose the event.</td>
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<tr>
<td>CCSS.MATH.CONTENT.7.NS.1 Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers.</td>
<td>CCSS.MATH.CONTENT.7.SP.C.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.</td>
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<tr>
<td>CCSS.MATH.CONTENT.6.RP.A.3</td>
<td>CCSS.MATH.CONTENT.7.SP.C.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.</td>
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<tr>
<td>CCSS.MATH.CONTENT.7.SP.C.7 Develop a probability model to find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.</td>
<td>CCSS.MATH.CONTENT.7.SP.C.7 Develop a probability model to find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.</td>
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<tr>
<td>Probability is not revisited in 8th grade. Next use is in high school geometry.</td>
<td><strong>Next Grade Standard</strong></td>
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</table>

Probability is not revisited in 8th grade. Next use is in high school geometry.
This is the first exposure with probability. Inside of probability 7th graders will learn to represent probability as a fraction, sum of fractions, product of fractions, decimals, percents, etc. Therefore they begin to apply the use of operations of decimals and fractions. In both 6th grade and 7th grade prior to this module, students are becoming fluent in translating between fraction, decimal, and percent representations of numbers.

Probability is not visited prior to grade 7 or revisited again in 8th. Students do not see this content again until high school geometry so we need to make sure we teach it well. 😊

**Anchor Problem:**

Have students discuss what games they like to play at the Greeley Stampede or other various carnivals or amusement parks. Why do they like certain games, why do some games have bigger or smaller prizes then others? Inquire if they believe they have the same chance of winning all the games or if others are created being more difficult to win. Students will be given the task of creating a carnival game. Each student will be responsible for providing directions to their game and what the theoretical probability of winning the game will be. On carnival day students will rotate through the games. The game creator is responsible for keeping track of the data as "customers" pass through. They will compile the data at the end of the day to find the experimental probability of winning their game and compare it to their theoretical probability. The "customers" rotating through the games will be responsible for finding and documenting the theoretical probability of winning the game before they play it. You create a game that has a spinner with 5 different colors and you want to know the probability of it landing on each of the colors, how would you go about solving that? How will
you develop your data table for you carnival game? (What will the rows and columns have inserted into them?) How will you find the theoretical probability of your carnival game? How will you analyze the differences between your experimental and theoretical probability for your carnival game? What information do you need to create a tree diagram for your carnival game? Have students then create a sample tree diagram with "fake" data. What would you need to add to your carnival game in order to make it a compound event? Should you add to your game to make it compound or should it stay as is? What would be the tradeoffs/negatives of your decision, positives? 

Continuation of Topic A: Students will be given the task of creating a carnival game. Each student will be responsible for providing directions to their game and what the theoretical probability of winning the game will be. On carnival day students will rotate through the games. The game creator is responsible for keeping track of the data as "customers" pass through. They will compile the data at the end of the day to find the experimental probability of winning their game and compare it to their theoretical probability. The "customers" rotating through the games will be responsible for finding and documenting the theoretical probability of winning the game before they play it. What research of your carnival game have you done or still need to do before we run the simulation? If you could redo your carnival game, what would you do differently? Would you have a different game, different outcomes, and another option for data collection?

Preparing the learner:
Be sure to give a pre-assessment to make sure students understand concept of percent and the ability to translate from fractions to decimals to percents.

Big ideas for this module:
- That probability is a chance something will happen between 0 (will not happen) to 1 (absolutely will happen).
- Experimental vs Theoretical probability
- Compound Probability
- Probability models

Recall/Skills-
- I can determine the sample space for an event.
- I can determine the theoretical probability of an event.
- I can determine the experimental probability of an event.
- I can determine the probability of compound events occurring.
Show all possible arrangements of the letters in the word FRED using a tree diagram. If each of the letters is on a tile and drawn at random, what is the probability that you will draw the letters F-R-E-D in that order? What is the probability that your “word” will have an F as the first letter?

The container below contains 2 gray, 1 white, and 4 black marbles. Without looking, if you choose a marble from the container, will the probability be closer to 0 or to 1 that you will select a white marble? A gray marble? A black marble? Justify each of your predictions.
Knowledge Packet for Grade 7 Module 5: Revised 8/10/16

Making Connections

- I can describe the difference between the probability of an event found through experiment or simulation versus the theoretical probability of that same event.
- I can describe changes in the experimental probability as the number of trials increases.

Charles says he just flipped a coin 3 times and got heads twice so he says the probability of getting heads is \( \frac{2}{3} \). Jaimie disagrees and says the probability is \( \frac{1}{2} \) because there is one chance to flip heads out of two possibilities. Who is correct and defend your position.

- Explain the relationship between experimental probability and theoretical probability? Give an example to support your explanation.

Teacher Ideas for Interaction

Eureka

In Topics A and B, students learn to interpret the probability of an event as the proportion of the time that the event will occur when a chance experiment is repeated many times (7.SP.C.5). They learn to compute or estimate probabilities using a variety of methods, including collecting data, using tree diagrams, and using simulations. In Topic B, students move to comparing probabilities from simulations to computed probabilities that are based on theoretical models (7.SP.C.6, 7.SP.C.7). They calculate probabilities of compound events using lists, tables, tree diagrams, and simulations (7.SP.C.8). There is a very nice simulation in L9 if you have students mature enough for it. Otherwise skip it. L10 has more concrete simulations and should be used. In L11 students explain random-number tables and how they work. They learn to use probabilities to make decisions and to determine whether or not a given probability model is plausible (7.SP.C.7). This lesson involved knowing the probabilities for outcomes of a sample space. You used these probabilities to determine whether or not the simulation supported a given theoretical probability. The simulated probabilities, or estimated probabilities, suggested a workable process for understanding the probabilities. Only 50 trials were conducted in some examples; however, as stated in several other lessons, the more trials that are observed from a simulation, the better. The Mid-Module Assessment follows Topic B. Teach these topics well. Topics C and D are NOT part of the priority standard. Teach these topics at the end of the year.
Knowledge Packet for Grade 7 Module 5: Revised 8/10/16

CCSS Math Resources
7.SP.6
7.SP.7
7.SP.8

Marble Mania
Random Drawing Tool

7.SP.C.8
NCTM Illuminations
• Adjacent Circles

7.SP.C.5/7.SP.C.6/7.SP.C.7
Inside Mathematics
• Fair Games Black & White
• Fair Games Color
• Got Your Number

MARS Shell Center
• Card Game
• Charity Fair
• Lottery
• Memory Game
• Spinner BINGO

Illustrative Mathematics
• Rolling Dice
• Heads or Tails
• Tossing Cylinders
• Rolling Twice
• Waiting Times

Inside Mathematics
• Friends You Can Count On
• On Balance
• Party Time
• Rod Trains
• Squirreling it Away
**Knowledge Packet for Grade 7 Module 5: Revised 8/10/16**

**NCTM Illuminations**
- SKUNK Game (C)

**Math’s Mate Skill Builder Blue/Green**
- 30.1-30.2 Describing and Recognizing the Likelihood of an Event

**Math’s Mate Skill Builder Blue/Green**
- 30.4 Finding the Possible Outcomes (Sample Space) of an Event by Completing Tables and Tree Diagrams

**Pre-Assessment Module 5**
**Post Assessment Module 5**
**Common Assessment Module 5**

**Representations:**
- Spinners, coins, marbles, color cubes, cards, or other manipulatives used for experimental probability
- Tree diagrams, charts
- Graphing calculator programs (probability simulations)

**Vocabulary:**
- Probability, Probability model, Uniform probability model, Compound event, Tree diagram, Simulation, Relative Frequency, Replacement

**Probing questions:**
- What is probability?
- What does it mean when a meteorologist says there is a 50/50 chance of rain tomorrow? How does that affect your day?
- How do you know if an event is impossible, unlikely, likely, or certain to occur?
- What does a probability of .87 mean for the event?
- What does a probability of 1 mean for the event?
- What does a probability of 0 mean for the event?
- What does it mean for a game to be “fair”?
- Explain what it means to be a “chance experiment”.
- If you are going to a horse race, what information/data would you want to know about the horses and the jockeys before you select a horse that you want to win?
- Explain the difference between “with replacement” and “without replacement” and how it affects your probability. Use an example to illustrate your explanation.
Why is the numerator of the fraction not always 1 in a collection of events? Give an example of probability where the numerator is NOT 1.

Compare and contrast finding the probability of events equally likely to occur vs events not equally likely to occur. Provide an example to support your explanation.

What is the purpose of a “tree diagram” and how is it helpful?

Describe a compound event. Provide an example to support your description.

How is the probability calculated different for events that do NOT occur equally and compound events? Provide examples to support your explanation.

Are all outcomes in an event equally likely to happen? Provide an example and counter example to illustrate your explanation.

Describe the process for calculating the probability of an event NOT happening. Provide an example to support your explanation.

Why would you want to know the experimental probability of an event if you have already correctly calculated the theoretical probability?

Can you think of any situations where the first stage of a tree diagram

What are some situations in life that would be considered a "compound event" for calculating probabilities? How do you know it is a compound event vs. what we worked on in previous lessons?

How is sample space related to the probability of a chance experiment? Choose an example to illustrate your explanation.

What similarities can you find between designing a probability simulation vs. a lab in science? Why do you think the two are so closely related?

Explain relative frequency and how we can use it to estimate probability. Provide an example to support you explanation.

How is relative frequency related to theoretical probability? Explain your thinking.
Focus Standard:

**Topic A:** Unknown Angles (7.G.B.5) (February 27 – March 2) 4 days

**Geography:** Area, Circumference, and Perimeter formulas for quadrilaterals, triangle, circles, as well as irregular areas. (4 days)

<table>
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<tr>
<td>CCSS.MATH.CONTENT.4.MD.7</td>
<td><strong>Focus Standards:</strong> CCSS.MATH.CONTENT.7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.</td>
<td>8.G.1 Verify experimentally the properties of rotations, reflections, and translations</td>
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<td>8.G.2 – Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.</td>
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<td>8.G.4 – Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.</td>
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<td>8.G.5 – Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.</td>
</tr>
</tbody>
</table>

**Changes**

The focus in 6th grade is on measuring angles and recognizing relationships between angles. They do write equations and solve simple angle measures. 7th graders focus on angle pairs such as: Supplementary, Complementary, Vertical, and Adjacent Angles

**Changes**

Students really need to be fluent in these 7th grade concepts because in 8th grade they begin looking heavily at rigid transformations and similar figures. Also 8th graders learn about the relationships of the angles formed by parallel lines cut by a transversal: alternate interior, alternate exterior, corresponding, same side supplemental, etc. 8th graders also spend quite a bit of time on rigid transformations which
# Knowledge Packet for Grade 7 Module 6 2016-17: Revised 8/10/16

## Anchor Problem:

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## Preparing the learner:
A pre-assessment should be given to find out if students can solve for the measures of angles using complementary, supplementary, vertical angles, etc. Topic A needs to be fluent. The remainder of the module is optional as students will revisit this in high school geometry.

## Big ideas for this topic:

- **Solving for measures of angles**

## Recall/Skills:

- I can use algebra to find the measure of angles.

## Making Connections

- I can infer things from the figure that I need to solve all angles.
In Topic A, students solve for unknown angles. The supporting work for unknown angles began in Grade 4, Module 4 (4.MD.C.5–7), where all of the key terms in this Topic were first defined, including: adjacent, vertical, complementary, and supplementary angles, angles on a line, and angles at a point. In Grade 4, students used those definitions as a basis to solve for unknown angles by using a combination of reasoning (through simple number sentences and equations), and measurement (using a protractor). For example, students learned to solve for a missing angle in a pair of supplementary angles where one angle measurement is known. *Because these are previously learned topics in topic A, I recommend giving a pretest and those who have already mastered get to move to a new area while those who have not mastered need to do topic A for review. This entire module is focus but topic A is pretty baseline. You could include problems from topics D and E since they are center around topics of area, surface area, or volume.* Remember everything in this module is focus not priority.

**Blended Resources, Personal Learning Resources, Differentiated Learning Resources**

**CCSS Math Resources**

**Common Core stations 6th grade**

**Quia (google quia solving for the measures of angles for jeopardy, rags to riches, matching, concentration, or quizzes)**

**Pre-Assessment Module 6 topic AB**

**Pre-Assessment Module 6 topic CD**

**Vocabulary:**

- **Correspondence** *(A correspondence between two triangles is a pairing of each vertex of one triangle with one and only one vertex of the other triangle. A triangle correspondence also induces a correspondence between the angles of the triangles and the sides of the triangles.)*

- **Identical Triangles** *(Two triangles are said to be identical if there is a triangle correspondence that pairs angles with angles of equal measure*
and sides with sides of equal length.)

- **Unique Triangle** (A set of conditions for two triangles is said to determine a *unique* triangle if whenever the conditions are satisfied, the triangles are identical.)
- **Three sides condition** (Two triangles satisfy the *three sides condition* if there is a triangle correspondence that pairs all three sides of one triangle with sides of equal length. The three sides condition determines a unique triangle.)
- **Two angles and the included side condition** (Two triangles satisfy the *two angles and the included side condition* if there is a triangle correspondence that pairs two angles and the included side of one triangle with angles of equal measure and a side of equal length. This condition determines a unique triangle.)
- **Two angles and the side opposite a given angle condition** (Two triangles satisfy the *two angles and the side opposite a given angle condition* if there is a triangle correspondence that pairs two angles and a side opposite one of the angles with angles of equal measure and a side of equal length. The two angles and the side opposite a given angle condition determines a unique triangle.)
- **Two sides and the included angle condition** (Two triangles satisfy the *two sides and the included angle condition* if there is a triangle correspondence that pairs two sides and the included angle with sides of equal length and an angle of equal measure. The two sides and the included angle condition determines a unique triangle.)
- **Two sides and a non-included angle condition** (Two triangles satisfy the *two sides and a non-included angle condition* if there is a triangle correspondence that pairs two sides and a non-included angle with sides of equal length and an angle of equal measure. The two sides and a non-included angle condition determines a unique triangle if the non-included angle measures 90° or greater. If the non-included angle is acute, the triangles are identical with one of two non-identical triangles.)
- **Right rectangular pyramid** (Given a rectangular region $B$ in a plane $E$, and a point $V$ not in $E$, the rectangular pyramid with base $B$ and vertex $V$ is the union of all segments $VP$ for any point $P$ in $B$. It can be shown that the planar region defined by a side of the base $B$ and the vertex $V$ is a triangular region, called a lateral face. If the vertex lies on the line perpendicular to the base at its center (the intersection of the rectangle's diagonals), the pyramid is called a right rectangular pyramid.)
- **Surface of a pyramid** (The *surface of a pyramid* is the union of its base region and its lateral faces.)
- **Congruence**: If two corresponding have the same measure (equal lengths or angles) the segments or angles are considered congruent. Numbers are equal, figures are congruent.
- Vertical angles – angles created by intersecting lines
- Adjacent angles – angles whom share a side (are next to each other)
- Complementary Angles – 2 angles that add up to 90 degrees
- Supplementary Angles- 2 angles that add up to 180 degrees
- Angles on a line – when all rays in all angles meet at the same point on the line, the sum of the angles will be 180 degrees
- Angles at a Point – the sum of all angles whose vertex is a point is 360 degrees
- Right rectangular prism -3D figure with 6 faces whose sides are perpendicular to both bases

**Probing questions:**
<table>
<thead>
<tr>
<th>Explain how angles on a line are related to Supplementary angles?</th>
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<tbody>
<tr>
<td>Complementary angles and angles around a point are related to each other how?</td>
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<tr>
<td>Describe the relationship between same side interior angles formed by parallel lines cut by a transversal and supplementary angles? Be sure to provide an example to illustrate your description.</td>
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<tr>
<td>In order to set up an equation to solve for the measure of angle, what do you need to know. Be sure to provide an example.</td>
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<td>How can you determine if your answer is appropriate?</td>
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</tbody>
</table>
Focus Standard:

<table>
<thead>
<tr>
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<tr>
<td>CCSS.MATH.CONTENT.6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.</td>
<td><strong>Focus Standards:</strong> CCSS.MATH.CONTENT.7.G.A.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle.</td>
<td>CCSS.MATH.CONTENT.8.G.A.5Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.</td>
</tr>
</tbody>
</table>

Changes

| 6th graders learn area by decomposing both regular and irregular shapes. 7th graders are constructing figures with a compass and a straightedge. | 7th graders spend quite a bit of time using algebra to help find the measures of angles and sides. 8th graders extend this to angles created by parallel lines cut by a transversal and properties of them. They will also use algebra to find the measures of these angles. |

Anchor Problem: Royal Gorge Bridge

The support of a bridge is triangles. How do we create those triangle supports? What do we need to know to create these

- What went into designing this suspense bridge?
- The supports look like squares, take a closer look
- How many triangle supports are needed for this suspense bridge?
- How can you determine what parts of triangles correspond to one another?
- If you do not have necessary materials to draw a geometric shape with given parameters, how could you do that?

Preparing the learner: This should be student’s first experience working with compass, protractor, and straight edge (as opposed to ruler). Prepare some time to have students practice these but pre-test not required. Students should know the difference between and angle and a side
Big ideas for this topic:

Constructing and identifying congruent figures

Recall/Skills:
- I can draw a triangle using a compass, straightedge, and given information

Making Connections
- I can use information provided to me to determine whether a unique triangle is possible

Teacher Ideas for Interaction

Eureka/EngageNY

Next, in Topic B, students work extensively with a ruler, compass, and protractor to construct geometric shapes, mainly triangles (7.G.A.2). The use of a compass is new (e.g., how to hold it, and to how to create equal segment lengths). Students use the tools to build triangles, provided given conditions, such side length and the measurement of the included angle (MP.5). Students also explore how changes in arrangement and measurement affect a triangle, culminating in a list of conditions that determine a unique triangle. Students understand two new concepts about unique triangles. They learn that under a condition that determines a unique triangle: (1) a triangle can be drawn and (2) any two triangles drawn under the condition will be identical. It is important to note that there is no mention of congruence in the CCSS until Grade 8, after a study of rigid motions. Rather, the focus of Topic B is developing students' intuitive understanding of the structure of a triangle. This includes students noticing the conditions that determine a unique triangle, more than one triangle, or no triangle (7.G.A.2). Understanding what makes triangles unique requires understanding what makes them identical.

Lesson 5 is so very important because this where they learn how to identify corresponding parts and the symbols that show congruence with ultimate goal of proving 2 figures congruent and what that means. Most of the rest of the topic is identifying the minimum number of parts you would have to do to prove congruence. Lessons 6-8 focus on (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on
constructing triangles from three measurements of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. Instruction including the use of a compass is included to develop deeper understanding of geometric relationships and also as preparation for high school geometry. Lesson 6 proposes a gallery walk but if you do make sure you have a focus that students have to respond to, reflect on, or other questions for them to help them identify the rules. In lesson 7 they have students make a T-square to construct parallelograms. You could easily skip this lesson. You could combine lessons 6-8 and turn them into stations.

Lessons 9-12 are then spent figuring out what are the minimum parts of a triangle you would have to construct in order to have an identical triangle. Spend some time having students figure it out before you just give them a rule. In contrast to Lesson 12, where students had to examine pairs of distinct triangles, the diagrams of triangles in Lesson 13 are presented so that a relationship exists between the triangles due to the way they are positioned. They may share a common side, may be arranged in a way so that two angles from the triangles are vertical angles, and so on. Students must use the structure of each diagram in order to establish whether a condition exists that renders the triangles identical.

Lessons 13-14 students used the minimum conditions to determine whether two figures are congruent (identical). Most of the figures share a common side or overlap. In Lesson 15, students continue to apply their understanding of the conditions that determine a unique triangle. In Lesson 14, they were introduced to diagrams of triangles that had pre-existing relationships, as opposed to the diagrams in Lesson 13 that showed distinct triangles with three matching, marked parts. This added a new challenge to the task of determining whether triangles were identical because some information had to be assessed from the diagram in order to establish a condition that would determine triangles as identical. In Lesson 15, students are exposed to yet another challenge where they are asked to determine whether triangles are identical and to show how this information can lead to further conclusions about the diagram (i.e., showing why a given point must be the midpoint of a segment). It is recommended that you chunk these sections and differentiate. It is also expected that most teachers will not get to topics D and E so you could throw a few of those problems in here for review. This module makes for good station rotation as long as there are mini teaches included each day.

**Blended Resources, Personal Learning Resources, Differentiated Learning Resources**

**CCSS Math Resources**

**Common Core stations 7th grade**

**Mars Shell Centre**

**Quia (google Quia congruent triangles for jeopardy, rags to riches, matching, concentration, or quizzes)**

**Post Assessment topic AB**

**Probing questions:**
- What does it mean to be congruent?
- Describe corresponding. Be sure to use an example to support your description.
- What does it mean when there are an equal number of tick marks on two corresponding sides?
Focus Standard:

Strand(s): Topic C: Slicing Solids (7.G.A.3) May 15-24 7 days

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<tbody>
<tr>
<td>CCSS.MATH.CONTENT.6.G.A.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.</td>
<td>Focus Standards: CCSS.MATH.CONTENT.7.G.A.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.</td>
<td>CCSS.MATH.CONTENT.8.G.C.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.</td>
</tr>
</tbody>
</table>

Changes

In 6th grade nets are first exposure as a way to find surface area of any 3D figure. So essentially they covered and filled 3D figures. Now in 7th grade they see them as solids and begin to understand what cross section.

In both 6th and 7th students work with surface area and volume of 3D figures that were right pyramids and prisms. In 8th grade this expands to surface area and volume of circular bases such as cones, cylinders, and spheres.

Anchor Problem:

Preparing the learner:
Students have no prior experience with cross sections which will provide baseline knowledge for conics in High School Geometry. If you do not have a set of solids that can be taken apart use playdough and plastic knives. Students can build the 3 day shape and then experiment with different cuts. Also make sure you have models ready for parallel planes and perpendicular planes.

Big ideas for this topic:

- Recognizing cross sections of 3D figures
Recall/Skills:
- I can describe the different type of slices of a 3D figure

What shapes can be created by one slice through a cube? Look for these possibilities:

a. a square
b. an equilateral triangle
c. a rectangle that is not a square
d. a triangle that is not equilateral
e. a pentagon
f. a hexagon
g. a parallelogram that is not a rectangle
h. an octagon
i. a circle

Making Connections
- I could draw or explain all cross sections for a 3D figure

Three vertical slices perpendicular to the base of the right rectangular pyramid are to be made at the marked locations: (1) through AB, (2) through CD, and (3) through vertex E. Based on the relative locations of the slices on the pyramid, make a reasonable sketch of each slice. Include the appropriate notation to indicate measures of equal length:

(1) Slice through AB
(2) Slice through CD
(3) Slice through vertex E

Teacher Ideas for Interaction

Eureka/EngageNY
- Students examine the cross sections of solid figures in the next four lessons. In Lessons 16 and 17, students examine slices made parallel or perpendicular to a face of a solid before moving to angled slices in Lesson 18. To help students visualize slices, provide them with the right
rectangular prism nets included after Lesson 27 (and later, the right rectangular pyramid nets) to build and refer to as they complete the lesson. In lesson 19, Students describe three-dimensional figures built from cubes by looking at horizontal slicing planes. *Again playdough is an excellent resource as well as cubes for Lesson 19. Students need to trial and error until they come to an understanding.*

**Blended Resources, Personal Learning Resources, Differentiated Learning Resources**

**CCSS Math Resources**

**Common Core stations 7th grade**

**25.10 Recognizing the Shapes of Cross Sections Through Three-Dimensional Shapes**

A site on Annenberg Learner that illustrates cross sections:

Quia (google Quia cross sections for jeopardy, rags to riches, matching, concentration, or quizzes)

**Post Assessment AB module 6**  
**Post Assessment CD module 6**  
**Common Assessment module 6**

**Probing questions:**

- How does a plane relate to a slice?
- How do two different people slice the same figure and get two different shapes for cross sections?
- How would you describe a pyramid?
- How is a rectangular pyramid different from a right rectangular pyramid?
- How can you relate a parallel slice to the base of a pyramid with scalars? Draw an example to support your explanation.
- Is it possible to get a triangle for a cross section of a right rectangular prism? Draw a pic to prove or disprove.
- Is it possible to get a pentagon for a cross section of a right rectangular prism? Draw a pic to prove or disprove.
- What shape will a cross section parallel to the base be in a right rectangular pyramid? Draw a pic to prove or disprove.
- Is there a rule for determining all possible shapes the cross sections can be?

It is not expected that teachers will get to topics D and E but should be familiar enough with them that they could pull problems for differentiation in topic A.