### Algebra I

**Relationships between quantities and reasoning and their graphs**

- **Topic A** Introduction to functions studied this year
- **Topic B** Structures of expressions
- **Topic C** Solving equations and inequalities
- **Topic D** System of equations

| Topic A – August 22 to August 25 | 4 days |
| Topic B – August 26 to September 7 | 8 days |
| Mid Module Assessment/Reteach | 5 days |
| September 8 to September 14 | 14 days |
| Topic C – September 15 to October 7 | 14 days |
| Topic D – October 10 to October 17 | 6 days |
| Review/Reteach/Assessment | 4 days |
| October 18 to October 21 | 4 days |

### Content Standards

**Reason quantitatively and use units to solve problems.**

- **N-Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

- **N-Q.2** Define appropriate quantities for the purpose of descriptive modeling.

- **N-Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**Create equations that describe numbers or relationships**

- **A-CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

- **A-CED.2** Create equations in two or more variable to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- **A-CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

- **A-CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law $V = IR$ to highlight resistance $R$.

**Understand solving equations as a process of reasoning and explain the reasoning**

- **A-REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

**Solve equations and inequalities in one variable**

- **A-REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

**Solve systems of equations**

- **A-REI.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

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### Algebra I

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-REI.6</td>
<td>Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.</td>
</tr>
<tr>
<td>A-REI.10</td>
<td>Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).</td>
</tr>
<tr>
<td>A-REI.12</td>
<td>Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</td>
</tr>
</tbody>
</table>

### Interpret the structure of expressions

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
</table>
| A-SSE.1  | Interpret expressions that represent a quantity in terms of its context.  
  
  a. Interpret parts of an expression, such as terms, factors, and coefficients.  
  
  b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret \( P(1+r)^n \) as the product of \( P \) and a factor not depending on \( P \). |
| A-SSE.2  | Use the structure of an expression to identify ways to rewrite it. For example, see \( x^4 - y^4 \) as \( (x^2 + y^2)(x^2 - y^2) \), thus recognizing it as a difference of squares that can be factored as \( (x^2 - y^2)(x^2 + y^2) \). |

### Perform arithmetic operations on polynomials

<table>
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<tr>
<th>Standard</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>A-APR.1</td>
<td>Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</td>
</tr>
</tbody>
</table>

### Video Links

- **Lesson 1 Elevation vs. Time 2**
  - [Video Link](http://www.mrmeyer.com/graphingstories1/graphingstories2.mov)
- **Lesson 1 Elevation vs. Time 3**
  - [Video Link](http://www.mrmeyer.com/graphingstories1/graphingstories3.mov)
  - [YouTube Video](https://youtu.be/xgODzAwxrx8)
- **Lesson 2 Ball rolling**
  - [Video Link](https://www.youtube.com/watch?v=ZCFBC8aXz-g)
- **Lesson 2 Elevations of time 4**
  - [Video Link](http://www.mrmeyer.com/graphingstories1/graphingstories4.mov)
- **Lesson 3 Bacteria**
  - [Video Link](https://www.youtube.com/watch?v=gEwzDydciWc)
- **Lesson 5 Stairs**
  - [Video Link](https://youtu.be/X9S6EvmCevI)

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*2016-2017*
**Assessment**

Possible Formative Assessments
- Mid-Module Assessment and Rubric
- End-of-Module Assessment and Rubric
- Teacher Created
- Exit Tickets
- SCA’s using Kuta software

Summative Assessment
- District Assessment: 2016-17 D6 AlgI Eureka Module 1 Common Assessment
- Teacher Created Assessment to address other standards

**Instructional Notes**

### Topic A: Introduction to Functions Studied this Year - Graphing Stories

#### Problems from Module 5: Lesson 1 Example 1

**Lesson 1 Graphs of Piecewise Linear Functions (Exploration Lesson)**

Student outcomes:
- Students define appropriate quantities from a situation (a graphing story), choose and interpret the scale and the origin for the graph, and graph the piecewise linear function described in the video. They understand the relationship between physical measurements and their representation on a graph.
  - NOTE states: "The man starts at "30 feet above the ground," which is clearly false. The students have no prior knowledge of the height of a flight/story of a building, which is 10 ft.

**Lesson 2 Graphs of quadratic functions (Exploration Lesson)**

Student outcomes:
- Students represent graphically a nonlinear relationship between two quantities and interpret features of the graph. They understand the relationship between physical quantities via the graph.
  - The lesson introduces function notation, however it is formally defined later in Module 3 Lesson 9.
  - \( h(t) \) models height as a function of time while \( h(x) \) models the trajectory of the same object. The two functions have some similarities and differences but it is not imperative that students understand these specific differences.
  - If using problem set #4 (c,d), recall Pythagorean Theorem.

**Lesson 3 Graphs of Exponential functions (Exploration Lesson)**

Student Outcomes:
- Students choose and interpret the scale on a graph to appropriately represent an exponential function. Students plot points representing the number of bacteria over time, given that bacteria grow by a constant factor over evenly spaced time intervals.

**Lesson 4 OMIT**
Lesson 5  Two Graphing stories (Exploration Lesson)
Student outcomes:
  ● Students interpret the meaning of the point of intersection of two graphs and use analytic tools to find its coordinates.
  ● Students develop the tools necessary to discern units for quantities in real-world situations and choose levels of accuracy appropriate to limitations on measurement. They refine their skills in interpreting the meaning of features appearing in graphs.
  ● **Students have little prior knowledge of dimensional analysis. An example, traveling 13mph for 6 minutes. This topic is included on the mid-module assessment therefore please address additional examples.

Topic B: The Structure of Expressions

Lesson 6  Algebraic expression distributive properties (Problem Set Lesson)
Student Outcomes:
  ● Students use the structure of an expression to identify ways to rewrite it.
  ● Students use the distributive property to prove equivalency of expressions.
    • Make sure you read through the "4-number game" to completely understand before lesson.
    • algebra tiles (2 dimension & 3 dimensions) would be helpful.

Lesson 7  Algebraic Expressions – Commutative/Associative (Socratic Lesson)
Student Outcomes:
  ● Students use the commutative and associative properties to recognize structure within expressions and to prove equivalency of expressions.
    • Do not skip over the picture representation of the properties - they are referred to in later lessons.
    • Problem set 10 refers to the rules of exponents-may have to review additional examples.

Lesson 8  Adding and subtracting polynomials (Socratic Lesson)
Students Outcomes:
  ● Students understand that the sum or difference of two polynomials produces another polynomial and relate polynomials to the system of integers; students add and subtract polynomials.
    • The concept of using a polynomial as place value when x = 10 is used throughout module 1.

Lesson 9  Multiplying polynomials (Problem Set Lesson)
Students Outcomes:
  ● Students understand that the product of two polynomials produces another polynomial; students multiply polynomials.
    • What do you notice about the terms along the diagonals in the rectangle you drew?
Mid Module Unit Assessment

Topic C: Solving Equations and Inequalities

Lesson 10 True/False equations (Problem Set Lesson)
Student Outcomes:

- Students understand that an equation is a statement of equality between two expressions. When values are substituted for the variables in an equation, the equation is either true or false. Students find values to assign to the variables in equations that make the equations true statements.

  - Important not to skip - lesson makes use of structure and understanding among equivalencies.

Lesson 11 Solutions sets equations and inequalities (Problem Set Lesson)
Student Outcomes:

- Students understand that an equation with variables is often viewed as a question asking for the set of values one can assign to the variables of the equation to make the equation a true statement. They see the equation as a “filter” that sifts through all numbers in the domain of the variables, sorting those numbers into two disjoint sets: the solution set and the set of numbers for which the equation is false.

- Students understand the commutative, associate, and distributive properties as identities (i.e., equations whose solution sets are the set of all values in the domain of the variables).

  - Important not to skip - concepts are used in future lessons.

  - Introduce set notation.

Lesson 12 Solution equation (Problem Set Lesson)
Student Outcomes:

- Students are introduced to the formal process of solving an equation: starting from the assumption that the original equation has a solution. Students explain each step as following from the properties of equality. Students identify equations that have the same solution set.

  - Solving quadratics using strategies developed in Lesson 10 & 11.

  - Lesson is about making use of structure with the properties when solving quadratics.

Lesson 13 Potential Danger (Problem Set Lesson)
Student Outcomes:

- Students learn if-then moves using the properties of equality to solve equations. Students also explore moves that may result in an equation having more solutions than the original equation.
Lesson 14 Solving inequalities (Problem Set Lesson)
Student Outcomes:
● Students learn if-then moves using the addition and multiplication properties of inequality to solve inequalities and graph the solution sets on the number line.
   ▪ Example 3 shows why the inequality sign switches direction using the addition property. This concept is referred to in the exit ticket.

Lesson 15 Solution set 2 + equations “and” “or” (Exploration Lesson)
Student Outcomes:
● Students describe the solution set of two equations (or inequalities) joined by either “and” or “or” and graph the solution set on the number line.
   ▪ After leading a discussion on compound mathematical statements using "and" and "or", have students think of scenarios that could be modeled using a compound inequality.

Lesson 16 Solving/Graphing inequalities “and” “or” (Problem Set Lesson)
Student Outcomes:
● Students solve two inequalities joined by “and” or “or” and then graph the solution set on the number line.

Lesson 17 Equations involving factored expressions (Socratic Lesson)
Student Outcomes:
● Students learn that equations of the form \((x-a)(x-b) = 0\) have the same solution set as two equations joined by “or”: \(x-a = 0\) or \(x-b = 0\). Students solve factored or easily factorable equations.
   ▪ This is not a time for teaching factoring. The solutions are more common sense. Factoring is taught in Module 4.

Lesson 18 Equations variable in denominator (Problem Set Lesson)
Student Outcomes:
● Students interpret equations like \(1/x = 3\) as two equations, \(1/x = 3\) and \(x \neq 0\), joined by “and.” Students find the solution set for this new system of equations.
   ▪ They are calling the restriction in domain as a compound inequality and writing a system of equations.

Lesson 19 Renaming formulas (Problem Set Lesson)
Student Outcomes:
● Students learn to think of some of the letters in a formula as constants in order to define a relationship between two or more quantities, where one is in terms of another, for example holding \(V\) in \(V=IR\) as constant and finding \(R\) in terms of \(I\).
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- Literal equations - equations with no numbers.

Lesson 20-21 Solution set equations/inequalities with two variables (Problem Set Lesson)
Student Outcomes:
- Students recognize and identify solutions to two-variable equations. They represent the solution set graphically. They create two-variable equations to represent a situation. They understand that the graph of the line $ax + by = c$ is a visual representation of the solution set to the equation $ax + by = c$.
- Students recognize and identify solutions to two-variable inequalities. They represent the solution set graphically. They create two-variable inequalities to represent a situation.
- Students understand that a half-plane bounded by the line $ax + by = c$ is a visual representation of the solution set to a linear inequality, such as $ax + by < c$. They interpret the inequality symbol correctly to determine which portion of the coordinate plane is shaded to represent the solution.

- Opening exercise on lesson 20 and lesson 21 is crucial.

Lesson 22-23 Solution set similar equations/inequalities (Problem Set Lesson, Exploration Lesson)
Student Outcomes:
- Students identify solutions to simultaneous equations or inequalities; they solve systems of linear equations and inequalities either algebraically or graphically.
- Students create systems of equations that have the same solution set as a given system.
- Students understand that adding a multiple of one equation to another creates a new system of two linear equations with the same solution set as the original system. This property provides a justification for a method to solve a system of two linear equations algebraically.

- High School has not taught elimination method in the past but needs to incorporate elimination in this lesson for simple systems.

Lesson 24 Application system equations/inequalities (Exploration Lesson)
Student Outcomes:
- Students use systems of equations or inequalities to solve contextual problems and interpret solutions within a particular context.

- Light version of linear programming.

Topic D: Creating equations to solve problems
If behind leave this topic out

Lesson 25 Solving problem in 2 ways (Modeling Cycle Lesson)

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Student Outcomes:
● Students investigate a problem that can be solved by reasoning quantitatively and by creating equations in one variable.
● Students compare the numerical approach to the algebraic approach.

Lesson 26-27 Recursive challenge problems (Modeling Cycle Lesson)
Student Outcomes:
● Students learn the meaning and notation of recursive sequences in a modeling setting.
● Following the modeling cycle, students investigate the double and add 5 game in a simple case in order to understand the statement of the main problem.
● Students learn the meaning and notation of recursive sequences in a modeling setting.
● Students use recursive sequences to model and answer problems.
● Students create equations and inequalities to solve a modeling problem.
● Students represent constraints by equations and inequalities and interpret solutions as viable or non-viable options in a modeling context.

- The recursive definition is only used in this lesson. Explicit/General definitions are NOT referred to in this lesson. (Explicit/General definition of sequences is covered in Module 3 Topic A).
- The restriction in domain starts to be addressed with recursive definition.
- Be careful that students do not develop the misunderstanding that recursive definitions are not just doubling and adding five.

Lesson 28 Federal income tax (Modeling Cycle Lesson)
Student Outcomes:
● Students create equations and inequalities in one variable and use them to solve problems.
● Students create equations in two or more variables to represent relationships between quantities and graph equations on coordinate axes with labels and scales.
● Students represent constraints by inequalities and interpret solutions as viable or non-viable options in a modeling context.

- This lesson addresses a Colorado Academic Standard for Personal Financial Literacy although it is not CCSS.

End of Module Assessment week of October 13
### Algebra

**Descriptive Statistics**

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<tr>
<th>Topic</th>
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<td>A</td>
<td>Shapes and Centers of Distributions</td>
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<tr>
<td>B</td>
<td>Describing Variability and Comparing Distributions</td>
</tr>
<tr>
<td>C</td>
<td>Categorical Data on Two Variables</td>
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<tr>
<td>D</td>
<td>Numerical Data on Two Variables</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Topic</th>
<th>Dates</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>October 24 to 25</td>
<td>2 days</td>
</tr>
<tr>
<td>B</td>
<td>October 26 (Lesson 8 only)</td>
<td>1 day</td>
</tr>
<tr>
<td>C</td>
<td>October 28 to October 31</td>
<td>2 days</td>
</tr>
<tr>
<td>D</td>
<td>November 1 to November 9</td>
<td>5 days</td>
</tr>
</tbody>
</table>

### Content Standards

**Summarize, represent, and interpret data on a single count or measurement variable.**

- **S-ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots).
- **S-ID.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
- **S-ID.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

**Summarize, represent, and interpret data on two categorical and quantitative variables.**

- **S-ID.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
- **S-ID.6** Represent data on two quantitative variables on a scatter plot and describe how the variables are related.
  - a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
  - b. Informally assess the fit of a function by plotting and analyzing residuals.
  - c. Fit a linear function for a scatter plot that suggests a linear association.

**Interpret linear models.**

- **S-ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- **S-ID.8** Compute (using technology) and interpret the correlation coefficient of a linear fit.
- **S-ID.9** Distinguish between correlation and causation

### Resources

**Possible Formative Assessments**
- Mid-Module Assessment and Rubric
- End-of-Module Assessment and Rubric
- Teacher Created
- Exit Tickets
- SCA’s using Kuta software

**Summative Assessment**
- District Assessment: 2016-17 D6 AlgI Eureka Module 2 Common Assessment

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**2016-2017**
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<thead>
<tr>
<th>Lesson</th>
<th>Topic A: Shapes and Centers of Distributions</th>
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<tbody>
<tr>
<td>Lesson 1</td>
<td>Distributions and Their Shape (Problem Set)</td>
</tr>
<tr>
<td>Student outcomes:</td>
<td>● Students have had some experience with shape and distribution back in 6th and 7th grade, but the depth of knowledge is more rigorous in this lesson than previously experienced and focuses on explanation</td>
</tr>
<tr>
<td>Lesson 2</td>
<td>Describing the Center of a Distribution (Exploration Lesson)</td>
</tr>
<tr>
<td>Student outcomes:</td>
<td>● This lesson provides an opportunity to revisit and address the modeling cycle previously introduced in Module 1, Topic D.</td>
</tr>
<tr>
<td></td>
<td>● The lesson emphasizes comparing mean and median to determine which is an appropriate measure of the center for a given distribution.</td>
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<td></td>
<td>● Use correct vocabulary such as approximate or about; for example: to say &quot;approximately symmetrical&quot; or &quot;the mean is about&quot; when appropriate.</td>
</tr>
<tr>
<td>Lesson 3</td>
<td>Estimating Centers and Interpreting the Mean as a Balance Point (Problem Set)</td>
</tr>
<tr>
<td>Student outcomes:</td>
<td>● The term balance point was not used in previous grades and will be an important component in lessons 4 - 6.</td>
</tr>
<tr>
<td>Topic B: Describing Variability and Comparing Distributions (OMIT Lessons 4, 5, 6 and 7)</td>
<td></td>
</tr>
<tr>
<td>Lesson 4</td>
<td>Summarizing Deviations from the Mean (Problem Set)</td>
</tr>
<tr>
<td>● Make sure students calculate the deviation from the mean, by subtracting the mean from the value, to reveal deviations above the mean (positive values) and deviations below the mean (negative values).</td>
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<tr>
<td>● As a result of the lesson, students should verbalize, the greater the variability (or spread) of the distribution, the greater the deviations from the mean.</td>
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<tr>
<td>Lesson 5</td>
<td>Measuring Variability for Symmetrical Distributions (Problem Set)</td>
</tr>
<tr>
<td>● Through experimentation, when calculating the sample standard deviation, dividing by (n-1) estimates the population standard deviation more accurately than dividing by n.</td>
<td></td>
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<tr>
<td>● It would be recommended to get together as a department and discuss this topic (dividing by n-1 compared to dividing by n).</td>
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</tr>
<tr>
<td>Lesson 6</td>
<td>Interpreting the Standard Deviation (Exploration Lesson)</td>
</tr>
<tr>
<td>● The TI calculator is used in this lesson to compute the standard deviation in order to compare the variability of data sets where the differences in variability are less obvious than in previous lessons.</td>
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<tr>
<td>Lesson 7</td>
<td>Measuring Variability for Skewed Distributions (Interquartile Range)(Exploration Lesson)</td>
</tr>
</tbody>
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- Students were previously exposed to the 5-number summary in 6th and 7th grades, but will need instruction on outliers, left-skewed and right-skewed.
- Left-skewed or skewed to the left means the data spreads out longer (like a tail) on the left side.
- Right-skewed or skewed to the right means the data spreads out longer (like a tail) on the right side.

**Lesson 8 Comparing Distributions (Exploration Lesson)**

Student Outcomes:
- An important point to make is that the median is used as a center of measure when a distribution is skewed or contains outliers, do not formally focus on spread.

**Topic C: Categorical Data on Two Variables (Could combine Lessons 9, 10 and 11)**

**Lesson 9 Summarizing Bivariate Categorical Data (Exploration Lesson)**

Student Outcomes:
- Categorical data is introduced in 6th grade and reviewed in 8th grade.

**Lesson 10 Summarizing Bivariate Categorical Data with Relative Frequencies (Exploration Lesson)**

Student Outcomes:
- Conditional relative frequency is the same as joint relative frequency referenced in the teacher materials.

**Lesson 11 Conditional Relative Frequencies and Association (Exploration Lesson)**

Student Outcomes:
- Challenge students to think critically about the meaning of the association between two categorical variables and be careful not to draw unwarranted conclusions about cause-and-effect relationships between two categorical variables.

**Topic D Numerical Data on Two Variables**

*Problems from Module 5: Lesson 1 Exit ticket; Lesson 1 PS 1-3; Lesson 2 Example 1; Lesson 3 PS 1*

**Lesson 12-13 Relationship between Two Numerical Variables (Problem Set)**

Student Outcomes:
- Scatter plots and linear regression models showing linear relationships are introduced in 8th grade.
- Vocabulary used in 8th grade is positive, negative and no correlation.
- Lessons 12 and 13 extend this concept to include exponential and quadratic models - focus: determine if relationship is linear or non-linear.

**Lesson 14 Modeling Relationships with a Line (Problem Set)**

Student Outcomes:
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- In 8th grade, students draw a line of best fit and approximate the line. Graphing calculators are not used in the 8th grade.
- OMIT residuals
- The regression line is calculated using the graphing calculator in this lesson.

**Lesson 15** Interpreting Residuals with a Line (OMIT)
- A residual is the actual y-value minus the predicted y-value which relates to the x-value minus the mean which was covered in lesson 4.

**Lesson 16** More on Modeling Relationships with a Line (OMIT)
**Student Outcomes:**
- A residual plot is used in this lesson.

**Lesson 17-18** Analyzing Residuals (OMIT)
- The graphing calculator is used to make residual plots to evaluate the accuracy of the best-fit line.

**Lesson 19** Interpreting Correlation (Problem Set)
**Student Outcomes:**
- Calculating the "r" value with calculator and interpreting the meaning
- Distinguish between correlation and causation

**Lesson 20** Analyzing Data Collected on Two Variables (Exploration Lesson)
**Student Outcomes:**
- Bring everything together. Referencing lesson 12 through 19. Creating an individual poster that analyzes two variables using all the tools previously used. ([www.amstat.org/education/posterprojects/index.cfm](http://www.amstat.org/education/posterprojects/index.cfm))
- A well-defined rubric is recommended; an example is available at [www.amstat.org/education/posterprojects/index.cfm](http://www.amstat.org/education/posterprojects/index.cfm).
### Algebra 1

#### Linear and Exponential Functions

**Topic A** Linear and Exponential Sequences
**Topic B** Functions and Their Graphs
**Topic C** Transformations of Functions
**Topic D** Using Functions and Graphs to Solve Problems

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<th>Length of Unit</th>
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<td>Review/Reteach/Assessment</td>
<td>December 14 to December 14</td>
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<td>Topic C – January 4 to January 20</td>
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<tr>
<td>Review/Reteach/Assessment</td>
<td>January 26 to January 27</td>
<td>2 days</td>
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#### Content Standards

**A-SSE.B.3** Write expressions in equivalent forms to solve problems.
- Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
  - Use the properties of exponents to transform expressions for exponential functions. For example, the expression $1.15^t$ can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

**A-CED.A.1** Create equations that describe numbers or relationships
- Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

**A-REI.D.11** Represent and solve equations and inequalities graphically
- Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

**F-IF.A.1** Understand the concept of a function and use function notation
- Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.

**F-IF.A.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

**F-IF.A.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.

**F-IF.B.4** Interpret functions that arise in applications in terms of the context
- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where...*
Algebra 1

<table>
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<th>Analyze functions using different representations</th>
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<tbody>
<tr>
<td><strong>F-IF.B.5</strong> Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <em>For example, if the function</em> ( h(n) ) <em>gives the number of person-hours it takes to assemble</em> ( n ) <em>engines in a factory, then the positive integers would be an appropriate domain for the function.</em></td>
</tr>
<tr>
<td><strong>F-IF.B.6</strong> Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Construct and compare linear and exponential models and solve problems</th>
</tr>
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<tbody>
<tr>
<td><strong>F-LE.A.1</strong> Distinguish between situations that can be modeled with linear functions and with exponential functions.</td>
</tr>
<tr>
<td>a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.</td>
</tr>
<tr>
<td>b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.</td>
</tr>
<tr>
<td>c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</td>
</tr>
<tr>
<td><strong>F-LE.A.2</strong> Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</td>
</tr>
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</table>

2016-2017
### Algebra 1

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-LE.A.3</td>
<td>Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.</td>
</tr>
<tr>
<td>F-LE.B.5</td>
<td>Interpret expressions for functions in terms of the situation they model</td>
</tr>
<tr>
<td></td>
<td>Interpret the parameters in a linear or exponential function in terms of a context.</td>
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</tbody>
</table>

#### Technology

<table>
<thead>
<tr>
<th>Topic</th>
<th>Video</th>
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<th>Description</th>
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<tbody>
<tr>
<td></td>
<td>Lesson 5</td>
<td>How folding paper can get you to the moon</td>
<td><a href="http://www.youtube.com/watch?v=AmFMJC45f1Q">http://www.youtube.com/watch?v=AmFMJC45f1Q</a></td>
</tr>
</tbody>
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#### Assessment

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<td>End-of-Module Assessment and Rubric</td>
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<td>Teacher Created</td>
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<td>Exit Tickets</td>
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<td>SCA’s using Kuta software</td>
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<table>
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<tr>
<th>Summative Assessment</th>
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<tr>
<td>District Assessment: 2016-17 D6 AlgI Eureka Module 3AB Common Assessment and 2016-17 D6 AlgI Eureka Module 3CD Common Assessment</td>
</tr>
<tr>
<td>Teacher Created Assessment to address other standards</td>
</tr>
</tbody>
</table>

#### Instructional Notes

**Topic A Linear and Exponential Sequences**

**Problems from Module 5: Lesson 3 Exit Ticket, Lesson 5 Exercise 1-3, Exit Ticket and Problem Set 1-4**

**Lesson 1 Integer Sequences (Problem Set)**

**Student Outcomes:**

- Students examine sequences and are introduced to the notation used to describe them.

**NOTE States:**

- Function notation is first covered in this lesson.
- In 8th grade students define, evaluate and compare functions NOT using function notation.
- Multiple representations of writing function notation.
- Introduction to function notation in this lesson without calling attention to it.
Lesson 2  Recursive Formulas for Sequences (Problem Set)
Student Outcomes:
- Students write recursive and explicit formulas for sequences

NOTE States:
- The recursive definition is synthesized with exponential and linear sequences.
- Students need to write the definition for the following sequences: 1, 2, 4, 8, 16, ... as $2^{n-1}$
- The focus is to obtain a high level understanding of recursive notation.

Lesson 3  Arithmetic and Geometric Sequences (Problem Set)
Student Outcomes:
- Students learn the structure of arithmetic and geometric sequences

NOTE States:
- Students differentiate between the two sequences.
- Write both explicit and recursive definitions of Arithmetic and Geometric sequences. “Lesson 4 Why Do Banks Pay YOU to Provide Their Services?”
- Generate the Simple Interest formula $I(t)=Prt$
- Generate the Compound Interest $FV = PV(1 + r)^t$

Lesson 4  Why do Banks Pay YOU to Provide Their Services? (Problem Set)
Student Outcomes:
- Students compare the rate of change for simple and compound interest and recognize situations in which a quantity grows by a constant percent rate per unit interval.

Lesson 5  The Power of Exponential Growth (Socratic Lesson)
Student Outcomes:
- Students are able to model with and solve problems involving exponential formulas.

NOTE States:
- Video “How folding paper can get you to the moon” (http://www.youtube.com/watch?v=AmFMJC45f1Q)

Lesson 6  Exponential Growth – US Population and World Population (Modeling)
Student Outcomes:
- Students compare linear and exponential models of population growth.

Lesson 7  Exponential Decay (Problem Set)
Student Outcomes:
- Students describe and analyze exponential decay models; they recognize that in a formula that models exponential decay, the growth factor $bb$ is less than 1; or equivalently, when $bb$ is greater than 1, exponential formulas with negative exponents could also be used to model decay
Lesson 8  Why Stay with Whole Numbers? (Problem Set)
Student Outcomes:

- Students use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- Students create functions that represent a geometric situation and relate the domain of a function to its graph and to the relationship it describes.

Lesson 9-10 Representing, Naming, and Evaluating Functions  (Socratic and Problem Set)
Student Outcomes:

- Students understand that a function from one set (called the domain) to another set (called the range) assigns each element of the domain to exactly one element of the range.
- Students use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- Students understand that a function from one set (called the domain) to another set (called the range) assigns each element of the domain to exactly one element of the range and understand that if \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \).
- Students use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

NOTE States:

- Read through lengthy explanation in lesson 9. This will aid in assessing what students know and need to know about function.
- Lesson 10 addresses domain and range in interval notation using brackets instead of inequalities.

Lesson 11  The Graph of a Function (Exploration)
Student Outcomes:

- Students understand set builder notation for the graph of a real-valued function.
- Students learn techniques for graphing functions and relate the domain of a function to its graph.

NOTE States:

- The pseudocode is not a necessary component of the lesson.

Lesson 12  The Graph of the Equation \( y = f(x) \) (Exploration)
Student Outcomes:

- Students understand the meaning of the graph of \( y = f(x) \), namely \( \{(x, y) \mid x \in D \text{ and } y = f(x)\} \). Students understand the definitions of when a function is increasing or decreasing.

NOTE States

- The pseudocode is not a necessary component of the lesson.

Lesson 13  Interpreting the Graph of a Function (Modeling Cycle)

2016-2017
Student Outcomes:
- Students create tables and graphs of functions and interpret key features including intercepts, increasing and decreasing intervals, and positive and negative intervals.

NOTE States
- Students create tables and graphs of functions and interpret key features including intercepts, increasing and decreasing intervals, and positive and negative intervals.

Lesson 14 Linear and Exponential Models – Comparing Growth Rates (Problem Set)
Student Outcomes:
- Students compare linear and exponential models by focusing on how the models change over intervals of equal length. Students observe from tables that a function that grows exponentially eventually exceeds a function that grows linearly.

NOTE States:
- The lesson stresses end behavior of a linear and exponential. An exponential function overtakes the linear as x goes to infinity.

**Alg 1 Eureka Module 3AB Common Assessment 2016-17**

** Semester 1 Final Exam**

- **Topic C Transformations of Functions**

  - **Graphing calculator is highly recommended for Topic C**

  - **Problems from Module 5: Lesson 1 Exercises 2-6**

Lesson 15 Piecewise Functions (Exploration)
Student Outcomes:
- Students examine the features of piecewise functions, including the absolute value function and step functions. Students understand that the graph of a function \( f \) is the graph of the equation \( y = f(x) \).

NOTE States:
- **Introduction and solving Absolute Value equation needs to added in this lesson.** Making the connection to the graph of the absolute value function and absolute value equations will help the students with their reasoning of the function. Absolute value equations are not solved in Algebra II.
- Graphing by hand.
- Students need to do the opening exercise in order to do the explore in the lesson.
- Introduction to the Greatest Integer function (Step Function).

Lesson 16 Graphs Can Solve Equations Too (Problem Set)

2016-2017
Student Outcomes:
● Students discover that the multi-step and exact way of solving \(|2x - 5| = |3x + 1|\) using algebra can sometimes be avoided by recognizing that an equation of the form \(f(x) = g(x)\) can be solved visually by looking for the intersection points of the graphs of \(y = f(x)\) and \(y = g(x)\).

NOTE States:
● Solving by graphing calculator with appropriate viewing window.
● Discuss the limitations of solving by graphing; refer to closing of lesson.

Lesson 17-20 Four Interesting Transformations of Functions (Exploration, Problem Set, Exploration, Exploration)
Student Outcomes:
● Students examine that a vertical translation of the graph of \(y = f(x)\) corresponds to changing the equation from \(y = f(x)\) to \(y = f(x) + k\).
● Students examine that a vertical scaling of the graph of \(y = f(x)\) corresponds to changing the equation from \(y = f(x)\) to \(y = kf(x)\).
● Students examine that a horizontal translation of the graph of \(y = f(x)\) corresponds to changing the equation from \(y = f(x)\) to \(y = f(x - k)\).
● Students examine that a horizontal scaling with scale factor \(k\) of the graph of \(y = f(x)\) corresponds to changing the equation from \(y = f(x)\) to \(y = f(x - k)\).
● Students apply their understanding of transformations of functions and their graphs to piecewise functions

NOTE States
● Lesson 17 - Vertical translating and vertical scaling
● Lesson 18 - Horizontal translation
● Lesson 19 - Horizontal scaling
● Lesson 20 - Applying transformations
● Eureka uses "k" exclusively to describe transformations instead of \(a\), \(h\) and \(k\).
● Students may struggle with the generalized notation \(f(x) + 3 & f(x - 6)\).
● Students will write piecewise functions - introduction in lesson 15.
● Transformations are revisited again in Module 4 Lessons 19 & 20.

Topic D Using Functions and Graphs to Solve Problems (Optional, if time permits)
Problems from Module 5: Lesson 2 Problem Set 1-4

Lesson 21 Comparing Linear and Exponential Models Again (Problem Set)
Student Outcomes:
● Students create models and understand the differences between linear and exponential models that are represented in different ways.

NOTE States:
● A suggestion with the opening exercise is to make cards of each square in the table and have students match in the correct columns.

Lesson 22 Modeling an Invasive Species Population (Modeling)
Student Outcomes:
● Students apply knowledge of exponential functions and transformations of functions to a contextual situation.
Lesson 23  Newton’s Law of Cooling (Modeling)
Student Outcomes:
● Students apply knowledge of exponential functions and transformations of functions to a contextual situation.
  NOTE States:
  ● At first, transformations do not seem relevant but then students need to describe Newton’s Law with transformations.

Lesson 24  Piecewise and Step Functions in Context (Modeling)
Student Outcomes:
● Students create piecewise and step functions that relate to real-life situations and use those functions to solve problems.
● Students interpret graphs of piecewise and step functions in a real-life situation.
  NOTE States
  ● Big exploratory project will take more than one day.
  ● Be sure to work each problem out to anticipate what students will struggle with and how long the lesson will take and how to group students.

**Alg 1 Eureka Module 3CD Common Assessment 2016-17**
Algebra I

### Content Standards

#### Polynomial and Quadratic Expressions, Equations, and Functions

**Topic A** Quadratic Expressions, Equations, Functions, and Their connection to rectangles

**Topic B** Using Different forms for Quadratic Functions

**Topic C** Functions Transformations and Modeling

**Use properties of rational and irrational numbers.**

**N-RN.B.3** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

**Interpret the structure of expressions.**

**A-SSE.A.1** Interpret expressions that represent a quantity in terms of its context.

- a. Interpret parts of an expression, such as terms, factors, and coefficients.
- b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret \( P(1 + r)n \) as the product of \( P \) and a factor not depending on \( P \).

**A-SSE.A.2** Use the structure of an expression to identify ways to rewrite it. For example, see \( x^2 - y^2 \) as \( (x^2)^2 - (y^2)^2 \), thus recognizing it as a difference of squares that can be factored as \( (x^2 - y^2)(x^2 + y^2) \).

**Write expressions in equivalent forms to solve problems.**

**A-SSE.B.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

- a. Factor a quadratic expression to reveal the zeroes of the function it defines.
- b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

**Perform arithmetic operations on polynomials.**

**A-APR.A.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

**Understand the relationship between zeros and factors of polynomials.**

**A-APR.B.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

**Create equations that describe numbers or relationships.**

**A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

**A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

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**2016-2017**
### Algebra I

**Solve equations and inequalities in one variable.**

A-REI.B.4 **Solve quadratic equations in one variable.**

- Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

- Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$.

**Represent and solve equations and inequalities graphically.**

A-REI.D.11 **Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.**

**Interpret functions that arise in applications in terms of the context.**

F-IF.B.4 **For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

F-IF.B.5 **Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.**

F-IF.B.6 **Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.**

**Analyze functions using different representations.**

F-IF.C.7 **Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.**

- Graph linear and quadratic functions and show intercepts, maxima, and minima.
- Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-IF.C.8 **Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.**

- Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F-IF.C.9 **Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.**

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2016-2017
Build new functions from existing functions.

**F-BF.B.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

**Technology**

Graphing Calculator

Video Lesson 10  Dolphins jumping  [http://www.youtube.com/watch?v=g8RoFdyYY3s](http://www.youtube.com/watch?v=g8RoFdyYY3s)

**Assessments**

- **Possible Formative Assessments**
  - Mid-Module Assessment and Rubric
  - End-of-Module Assessment and Rubric
  - Teacher Created
  - Exit Tickets
  - SCA’s using Kuta software

- **Summative Assessment**
  - District Assessment: 2016-17 D6 AlgI Eureka Module 4 Common Assessment
  - Teacher Created Assessment to address other standards

**Instructional Notes**

**Topic A: Quadratic Expressions, Equations, Functions, and Their Connection to Rectangles**

**Problems from Module 5: Lesson 3 Problem Set 3**

**Lesson 4 Problem Set 1 -2**

**Lessons 1–2: Multiplying and Factoring Polynomial Expressions (Problem Set Lesson)**

**Student outcomes:**

- **L1:** Students use the distributive property to multiply a monomial by a polynomial and understand that factoring reverses the multiplication process.
- **L1:** Students use polynomial expressions as side lengths of polygons and find area by multiplying.
- **L1:** Students recognize patterns and formulate shortcuts for writing the expanded form of binomials whose expanded form is a perfect square or the difference of perfect squares.

2016-2017
Algebra I

- L2: Students understand that factoring reverses the multiplication process as they find the linear factors of basic, factorable quadratic trinomials.
  - When students are pulling out common factors, differentiate between factoring and factoring completely to help students make connection of the concept of GCF.
  - Algebra Tiles could be used in lesson 2.

Lessons 3–4: Advanced Factoring Strategies for Quadratic Expressions (Problem Set Lesson)
Student outcomes:
- L3: Students develop strategies for factoring quadratic expressions that are not easily factorable, making use of the structure of the quadratic expression.
- L4: Students factor quadratic expressions that cannot be easily factored and develop additional strategies for factorization, including splitting the linear term, using graphing calculators, and using geometric or tabular models.
  - Having a discussion with your department about whether the scaffolding on page 45 is how this lesson should be presented or another technique would be appropriate.

Lesson 5: The Zero Product Property (Exploration Lesson)
Student outcomes:
- Students solve increasingly complex one-variable equations, some of which need algebraic manipulation, including factoring as a first step and using the zero product property.

Lesson 6: Solving Basic One-Variable Quadratic Equations (Problem Set Lesson)
Student outcomes:
- Students use appropriate and efficient strategies to find solutions to basic quadratic equations.
- Students interpret the verbal description of a problem and its solutions in context and then justify the solutions using algebraic reasoning.

Lesson 7: Creating and Solving Quadratic Equations in One Variable (Problem Set Lesson)
Student outcomes:
- Students interpret word problems to create equations in one variable and solve them (i.e., determine the solution set) using factoring and the zero product property.

Lesson 8: Exploring the Symmetry in Graphs of Quadratic Functions (Exploration Lesson)
Student outcomes:
- Students examine quadratic equations in two variables represented graphically on a coordinate plane and recognize the symmetry of the graph. They explore key features of graphs of quadratic functions: y-intercept and x-intercept, the vertex, the axis of symmetry, increasing and decreasing intervals, negative and positive intervals, and end behavior. They sketch graphs of quadratic functions as a symmetric curve with a highest or lowest point corresponding to its vertex and an axis of symmetry passing through the vertex.
  - In 8th grade students evaluate $y = x^2$, learn to sketch the graph, identify y-intercept, and identify increasing and decreasing intervals of the graph.
Lesson 9: Graphing Quadratic Functions from Factored Form, \( f(x) = a(x - m)(x - n) \) (Problem Set Lesson)
Student outcomes:
- Students use the factored form of a quadratic equation to construct a rough graph, use the graph of a quadratic equation to construct a quadratic equation in factored form, and relate the solutions of a quadratic equation in one variable to the zeros of the function it defines.
- Students understand that the number of zeros in a polynomial function corresponds to the number of linear factors of the related expression and that different functions may have the same zeros but different maxima or minima.
  ○ Typo on page 91 => on top of page the equation should be \( y = x^2 + 6x - 40 \).

Lesson 10: Interpreting Quadratic Functions from Graphs and Tables (Problem Set Lesson)
Student outcomes:
- Students interpret quadratic functions from graphs and tables: zeros (\( x \)-intercepts), \( y \)-intercept, the minimum or maximum value (vertex), the graph’s axis of symmetry, positive and negative values for the function, increasing and decreasing intervals, and the graph’s end behavior.
- Students determine an appropriate domain and range for a function’s graph and when given a quadratic function in a context, recognize restrictions on the domain.
  ○ http://www.youtube.com/watch?v=g8RoFdyYY3s

Topic B: Using Different Forms for Quadratic Functions
**Graphing calculator is highly recommended for Topic B**

Lessons 11–12: Completing the Square (Exploration Lesson, Problem Set Lesson)
Student outcomes:
- L11: Students rewrite quadratic expressions given in standard form, \( ax^2 + bx + c \) (with \( a = 1 \)), in the equivalent completed-square form, \( a(x - h)^2 + k \), and recognize cases for which factored or completed-square form is most efficient to use.
- L12: Students rewrite quadratic expressions given in standard form, \( ax^2 + bx + c \) (with \( a \neq 1 \)), as equivalent expressions in completed-square form, \( a(x - h)^2 + k \). They build quadratic expressions in basic business application contexts and rewrite them in equivalent forms.
  ○ Lesson 11 involves completing the square when \( a = 1 \).
  ○ Lesson 12 involves completing the square when \( a \) is not equal to 1 or 0.
  ○ Specific vocabulary to Lesson 12: Unit Price, Quantity, Revenue and Unit cost, Production Cost.

Lesson 13: Solving Quadratic Equations by Completing the Square (Problem Set Lesson)
Student outcomes:
- Students solve complex quadratic equations, including those with a leading coefficient other than 1, by completing the square. Some solutions may be irrational. Students draw conclusions about the properties of irrational numbers, including closure for the irrational number system under various operations.
  ○ Review irrational and rational numbers.

Lesson 14: Deriving the Quadratic Formula (Problem Set Lesson)
### Algebra I

**Student outcomes:**
- Students derive the quadratic formula by completing the square for a general quadratic equation in standard form, \( ax^2 + bx + c = 0 \), and use it to verify the solutions for equations from the previous lesson for which they have already factored or completed the square.
  - The lesson involves deriving quadratic formula as well as comparing the three different ways of solving a quadratic.

**Lesson 15: Using the Quadratic Formula (Problem Set Lesson)**

**Student outcomes:**
- Students use the quadratic formula to solve quadratic equations that cannot be easily factored.
- Students understand that the discriminant, \( b^2 - 4ac \), can be used to determine whether a quadratic equation has one, two, or no real solutions.
  - Imaginary Numbers/solutions are NOT talked about; only refer to "no real solution".
  - Relate solutions to x-intercepts of the graph.

**Lesson 16: Graphing Quadratic Equations from the Vertex Form, \( y = a(x - h)^2 + k \) (Exploration Lesson)**

**Student outcomes:**
- Students graph simple quadratic equations of the form \( y = a(x - h)^2 + k \) (completed-square or vertex form), recognizing that \((h, k)\) represents the vertex of the graph and use a graph to construct a quadratic equation in vertex form.
- Students understand the relationship between the leading coefficient of a quadratic function and its concavity and slope and recognize that an infinite number of quadratic functions share the same vertex.
  - Graphing calculator will speed the lesson up.
  - This lesson ties in vertical translations of a quadratic and the solutions to the equation.
  - Recognizing the vertex is the maximum or the minimum of the quadratic.

**Lesson 17: Graphing Quadratic Functions from the Standard Form, \( f(x) = ax^2 + bx + c \) (Problem Set Lesson)**

**Student outcomes:**
- Students graph a variety of quadratic functions using the form \( f(x) = ax^2 + bx + c \) (standard form).
- Students analyze and draw conclusions about contextual applications using the key features of a function and its graph.
  - General strategy for graphing a quadratic in standard form (page 175).

### Topic C: Function Transformations and Modeling

**Problems from Module 5: Lesson 6- All (Modeling from data)**
- Lesson 7- All (Modeling from data)
- Lesson 8-All (Modeling from verbal description)
- Lesson 9-All (Modeling from verbal description)

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2016-2017
Lesson 18: Graphing Cubic, Square Root, and Cube Root Functions (Exploration Lesson)
Student outcomes:
- Students compare the basic quadratic (parent) function, \( y = x^2 \), to the square root function and do the same with cubic and cube root functions. They then sketch graphs of square root and cube root functions, taking into consideration any constraints on the domain and range.
  - Engage NY stresses the relationship between inverses but uses the word "reverse" instead of inverse. The lesson discusses the graphs of "reverse" functions as being a reflection over the line \( y = x \).
  - The students use patterns in tables and graph to generalize the reverse functions that reflect over the line \( y = x \). There is an emphasis on the domain and range as well as the restriction in domain and range for the square root function.

Lesson 19: Translating Functions (Problem Set Lesson)
Student outcomes:
- Students recognize and use parent functions for linear, absolute value, quadratic, square root, and cube root functions to perform vertical and horizontal translations. They identify how the graph of \( y = f(x) + k \) relates to the graphs of \( y = f(x + k) \) for any specific values of \( k \), positive or negative, and find the constant value, \( k \), given the parent functions and the translated graphs. Students write the function representing the translated graphs.

Lesson 20: Stretching and Shrinking Graphs of Functions (Exploration Lesson)
Student outcomes:
- Students recognize and use parent functions for absolute value, quadratic, square root, and cube root to perform transformations that stretch and shrink the graphs of the functions. They identify the effect on the graph of \( y = f(x) \) when \( f(kx) \) and \( f(kx + k) \) for any specified value of \( k \), positive or negative, and identify the constant value, \( k \), given the graphs of the parent functions and the transformed functions. Students write the formulas for the transformed functions given their graphs.

Lesson 21: Transformations of the Quadratic Parent Function, \( f(x) = x^2 \) (Problem Set Lesson)
Student outcomes:
- Students make a connection between the symbolic and graphic forms of quadratic equations in the completed-square (vertex) form. They efficiently sketch a graph of a quadratic function in the form, \( f(x) = a(x - h)^2 + k \), by transforming the quadratic parent function, \( f(x) = x^2 \), without the use of technology. They then write a function defined by a quadratic graph by transforming the quadratic parent function.
  - This lesson is done without a graphing calculator.

Lesson 22: Comparing Quadratic, Square Root, and Cube Root Functions Represented in Different Ways (Exploration Lesson)
Student outcomes:
- Students compare two different quadratic, square root, or cube root functions represented as graphs, tables, or equations. They interpret, contextualize, and abstract various scenarios to complete the comparative analysis.
  - This is a high powered lesson. Make sure to devote adequate time for this lesson.

Lessons 23–24: Modeling with Quadratic Functions (Modeling Cycle Lesson, Problem Set Lesson)

2016-2017
Student outcomes:

- **L23:** Students write the quadratic function described verbally in a given context. They graph, interpret, analyze, check results, draw conclusions, and apply key features of a quadratic function to real-life applications in business and physics.
- **L24:** Students create a quadratic function from a data set based on a contextual situation, sketch its graph, and interpret both the function and the graph in context. They answer questions and make predictions related to the data, the quadratic function, and graph.
  - Both of these lessons will give a good indicator of the success on CMAS.
  - In lesson 24, the students take three points and use systems of equations to create a quadratic equation.

**End of Module Assessment: 2016-17 D6 Alg I Eureka Module 4 Common Assessment**